Solutions (Detailed)

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Modern Macroeconomics

ISCTE - IUL

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Problem A: Ricardian Equivalence

1. "Ricardian Equivalence means .... (see slides)

2.
$$g_1 = b_1 \qquad \text{———— 1st period constraint}$$

$$g_2 = \tau_2(a_1 r_1) - b_1(1+r_1) \qquad \text{—— 2nd period constraint}$$

cancelling out b_1 above leads to

$$g_1 + \frac{g_2}{1+r_1} = \frac{\tau_2(a_1 r_1)}{1+r_1}$$

Intertemporal ~~b~~budget constraint of the Government

3. The Lagrangian function can be written as

$$\mathcal{L} = \ln c_1 + \ln c_2 + \lambda\left[y_1 + \frac{y_2}{1+(1-\tau_2)r_1} - c_1 - \frac{c_2}{1+(1-\tau_2)r_1}\right]$$

Now, taking first order conditions with respect to c_1, c_2 and λ leads to (notice that $\beta = 1$):

$$\frac{\partial \mathcal{L}}{\partial c_1} = 0 \Rightarrow \frac{1}{c_1} = \lambda \qquad \text{(FOC 1)}$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = 0 \Rightarrow \frac{1}{c_2} - \frac{\lambda}{1+(1-\tau_2)r_1} = 0 \qquad \text{(FOC 2)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow y_1 + \frac{y_2}{1+(1-\tau_2)r_1} - c_1 - \frac{c_2}{1+(1-\tau_2)r_1} = 0 \qquad \text{(FOC 3)}$$

From the two first FOC we can obtain the Euler equation as

$$c_2 = \left[1+(1-\tau_2)r_1\right] c_1 .$$

Yes, the Euler Equation <u>does</u> depend upon taxes.

4. No, Ricardian Equivalence does not hold in this economy, because taxes do affect the optimal decisions of private agents:

J_2 affects c_1^*, c_2^* and, therefore, ~~also~~ also a_1^*.

Problem B

The only difference between the details of this particular problem and the material covered in classes has to do with this fact:

→ in classes, the social contribution was determined with a proportional tax upon income: $J \cdot Y_t$.

→ in this particular exercise, this contribution (we are told) is determined by a lump-sum tax: J_t^Y.

Apart from this particular difference, everything else is exactly the same (in terms of the steps that should be followed).

1. We know that in a fully funded system, taxes upon the young will not affect the optimal levels of consumption (c^y_t, c^o_{t+1}). Let's show this in this particular problem.

From the period t and period $t+1$ constraints of the private agents

$$c^y_t + s_t = y_t - J^y_t$$

$$c^o_{t+1} = (1+r)s_t + \underbrace{(1+r)J^y_t}_{\text{social benefits}}$$

we obtain the intertemporal consolidated budget constraint by cancelling s_t in both eq. above:

$$c^y_t + \frac{c^o_{t+1}}{1+r} = y_t \quad . \qquad (B.1)$$

Now, the Lagrangian can be written as

$$\mathcal{L} = \ln c_t^y + \ln c_{t+1}^o + \lambda \left[y_t - c_t^y - \frac{c_{t+1}^o}{1+r} \right]$$

Obtaining the first order conditions (FOC)

$$\frac{\partial \mathcal{L}}{\partial c_t^y} = 0 \Rightarrow \frac{1}{c_t^y} - \lambda = 0 \qquad \text{(FOC1)}$$

$$\frac{\partial \mathcal{L}}{\partial c_{t+1}^o} = 0 \Rightarrow \frac{1}{c_{t+1}^o} - \frac{\lambda}{1+r} = 0 \qquad \text{(FOC2)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow y_t - c_t^y - \frac{c_{t+1}^o}{1+r} = 0 \qquad \text{(FOC3)}$$

From the two first FOC we obtain that

$$c_{t+1}^o = (1+r)\, c_t^y . \qquad \text{(B.2)}$$

Now, using (B.2) and (FOC3) we get

$$c^o_{t+1} = (1+r)\left[y_t - \frac{c^o_{t+1}}{1+r}\right]$$

$$c^o_{t+1} = (1+r)\, y_t - \cancel{(1+r)}\frac{c^o_{t+1}}{\cancel{1+r}}$$

$$2\, c^o_{t+1} = (1+r)\, y_t$$

$$c^o_{t+1} = \frac{(1+r)}{2}\, y_t \, . \qquad (B.3)$$

Once the optimal level for c^o_{t+1} has been obtained, it is immediate to obtain the optimal level for c^y_t: just use (B.3) and (B.2). Then

$$c^y_t = \frac{c^o_{t+1}}{1+r} = \frac{\left[\cancel{(1+r)}/2\right]}{\cancel{1+r}}\, y_t = \frac{1}{2}\, y_t \, .$$

Therefore, we can conclude that the optimal levels of c^y_t and c^o_{t+1} are not affected by the levels of taxes (lump-sum in this case) that are imposed in order to finance the fully funded social security system.

2. If J_t^y does not affect the level of welfare — because the levels of consumption are unnaffected — then the optimal level of J_t^y is <u>irrelevant</u>: any level of J_t^y is compatible with the optimal level of wellfare.

3. In a PAYG system, the growth rate of the population is a key element. The budget contraint: (such that the PAYG social securiry system runs a balanced budget)

$$L_t \, J_t^y = b \cdot L_{t-1}$$

$$\underbrace{\left(L_t / L_{t-1}\right)}_{=(1+n)} J_t^y = b$$

$$b = \underbrace{(1+n)\, J_t^y}_{\text{social benefits per person}} .$$

Now, we can write down the constraints for the private agent, for each period of time, in the same way as we did for the case of a fully funded system. That is, for a PAYG system, these are

$$c_t^y + s_t = y_t - J_t^y$$

$$c_{t+1}^o = (1+r)s_t + \underbrace{(1+n)J_t^y}_{\text{social benefits}}$$

From which, we can derive the intertemporal consolidated constraint

$$c_t^y + \frac{c_{t+1}^o}{1+r} = y_t - J_t^y + \left(\frac{1+n}{1+r}\right) J_t^y \, . \qquad \text{(B.4)}$$

writting down the lagrangian

$$\mathcal{L} = \ln c_t^y + \ln c_{t+1}^o + \lambda\left[y_t - J_t^y + \underbrace{\left(\frac{1+n}{1+r}\right)}_{\phi} J_t^y - c_t^y - \frac{c_{t+1}^o}{1+r}\right]$$

First order conditions ... then the Euler Equation ... leads to

$$c^o_{t+1} = (1+r)\,c^y_t \, . \qquad (B.5)$$

Now, using (B.5) and (B.4) we obtain the optimal levels of consumption c^y_t and c^o_{t+1} as

$$\left.\begin{aligned} c^y_t &= \frac{1}{2}\left[y - J^y_t + \phi J^y_t\right] \\ c^o_{t+1} &= \frac{(1+r)}{2}\left[y - J^y_t + \phi J^y_t\right] \end{aligned}\right\} \text{ with } \phi = \frac{1+n}{1+r}$$

Therefore, it is easy to see that

$$\frac{\partial c^y_t}{\partial J_t} = \frac{1}{2}\left[\phi - 1\right] \quad > 0 \quad \text{if } n > r$$

$$\frac{\partial c^o_{t+1}}{\partial J_t} = \frac{(1+r)}{2}\left[\phi - 1\right] > 0 \quad \text{if } n > r.$$

4. The answer is: $\underline{\underline{n > r}}$.

Problem c

1. $\phi = 1$; u as important as π for the Central Bank.

$\phi > 1$; u more important than π

$\phi < 1$; u less " " "

$\phi = 0$; Central Bank cares only about π

2. $$\mathcal{L} = \phi[-k - \alpha(\pi - \pi^e)] + \pi^2$$

$$\frac{\partial \mathcal{L}}{\partial \pi_d} = 0 \Rightarrow -\alpha\phi + 2\pi_d = 0$$

$$\pi_d = \frac{\alpha\phi}{2} = \frac{0.1 \times 0.4}{2} = 0.02.$$

3. With $\pi^e = \pi$, and inserting this into the Loss function leads to

$$\mathcal{L} = -\alpha\phi + \pi^2$$

then

$$\frac{\partial \mathcal{L}}{\partial \pi_c} = 0 \Rightarrow 2\pi_c = 0$$

$$\pi_c = 0.$$

4. If $\phi = 0$, even under discretion we will obtain zero inflation:

$$\pi_d = \frac{\alpha \phi}{2} = \frac{0.1 \times 0}{2} = 0.$$