Inequality

— Week 6 —

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Summary

1. The problem
2. Recapitulating the equilibrium with no heterogeneity
3. Heterogeneity: case 1
4. Heterogeneity: case 2
5. The social planner case
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I – The problem

The problem of heterogeneity

1. Up to now, we have assumed:
   1. That all agents are identical
   2. If they are not identical (for example, if one had negative savings of -100) why should another agent exist with exactly positive savings of $+100$?
2. Completely unrealistic framework
3. We have to introduce some heterogeneity to explain the problem above?
4. On the aggregate: at the level of society, the aggregate level of savings has to be

$$0$$

The heterogeneity considered here: income inequality

1. We focus on heterogeneity in endowments: **income inequality**
2. Questions:
   1. How does one agent having more income affect the welfare of other agents?
   2. How would a "social planner" reallocate resources in a world of income inequality to maximize overall welfare?

The framework

1. Two types of agents, $i = 1, 2$.
2. Each type of agent has N elements
3. Only two periods: $t, t+1$
4. Each type of agent endowed with exogenous income stream

$$Y_{i,t}, Y_{i,t+1}$$

5. Each type can borrow or save at the interest rate

$$r_t$$

6. Standard consumption-saving problem for each type of household

The equilibrium at the aggregate level

1. Total or aggregate "demand" (expenditure) is given

$$Y_t^d = N_1 \cdot C_{1,t} + N_2 \cdot C_{2,t}$$

2. Total "supply" (endowment) is

$$Y_t^s = N_1 \cdot Y_{1,t} + N_2 \cdot Y_{2,t}$$

3. In equilibrium, the real interest rate (r_t) will adjust so that Total "demand" must equal Total "supply"

$$Y_t^d = Y_t^s$$

4. Total saving must be zero; saving of one type must equal borrowing of the other type

$$N_1 \cdot S_{1,t} + N_2 \cdot S_{2,t} = 0 \Rightarrow N_1 S_{1,t} = -N_2 S_{2,t}$$

II – Recapitulating the equilibrium with no heterogeneity

Recapitulating: equilibrium with no heterogeneity

1. Log utility of both types of agents

$$U(C_{i,t}) = \ln C_{i,t}$$

2. The standard intertemporal optimization problem for each individual agent is

$$\begin{aligned} & \max_{C_{i,t},C_{i,t+1}} \ln \ln C_{i,t} + \beta \ln C_{i,t} \\ & \textit{subject to} \\ C_{i,t} + \frac{C_{i,t+1}}{1+r_t} \quad = \quad & Y_{1,t} + \frac{Y_{i,t+1}}{1+r_t} \end{aligned}$$

3. The Euler equation gives our already well known result

$$C_{i,t+1} = \beta(1+r_t)C_{i,t}$$

Recapitulating: equilibrium with no heterogeneity (cont.)

1. The optimal level of consumption is given by

$$C_{i,t} = \frac{1}{1+\beta}\left(Y_{1,t} + \frac{Y_{i,t+1}}{1+r_t}\right)$$

2. Consider the following example

$$\begin{aligned} N_1 &= N_2 = N \\ \beta &= 0.9 \\ (Y_{1,t}, Y_{1,t+1}) &= (1, 1) \\ (Y_{2,t}, Y_{2,t+1}) &= (1, 1) \end{aligned}$$

Recapitulating: equilibrium with no heterogeneity (cont.)

1. This will lead to the following results

$$
\begin{aligned}
C_{1,t} &= \frac{1}{1+\beta}\left(1+\frac{1}{1+r_t}\right) \\
C_{2,t} &= \frac{1}{1+\beta}\left(1+\frac{1}{1+r_t}\right)
\end{aligned}
$$

2. Total demand in the economy is then

$$
\begin{aligned}
Y_t^d &= N \cdot C_{1,t} + N \cdot C_{2,t} \\
&= 2N\left(\frac{1}{1+\beta}\left(1+\frac{1}{1+r_t}\right)\right)
\end{aligned}
$$

3. Total supply in this economy is

$$
Y_t^s = 2N
$$

Recapitulating: equilibrium with no heterogeneity (cont.)

1. Equate demand and supply

$$2N\left(\frac{1}{1+\beta}\left(1+\frac{1}{1+r_t}\right)\right)=2N$$

2. Leads to the equilibrium level of the interest rate

$$r_t=\frac{1}{\beta}-1$$

3. Now, plug this in to the consumption functions

$$\begin{aligned} C_{1,t} &= \frac{1}{1+\beta}(1+\beta)=1 \\ C_{2,t} &= \frac{1}{1+\beta}(1+\beta)=1 \end{aligned}$$

Recapitulating: equilibrium with no heterogeneity (cont.)

1. And, for consumption at $t+1$ we get

$$C_{1,t+1} = C_{2,t+1} = 1$$

2. In equilibrium, each household ends up consuming their endowment each period.
3. Notice that for both types of agents utility will be

$$U = \ln(1) + 0.9\ln(1) = 0.$$

4. Notice that utility is an ordinal concept: utility of 0 doesn't mean zero satisfaction.

III – Heterogeneity: case 1

— Temporarily Rich Type 2 —

Temporarily Rich Type 2

1. Now, let's change the setup in the following way
2. Type 1 households still have the same endowment pattern

$$(Y_{1,t}, Y_{1,t+1}) = (1, 1)$$

3. But the type 2 agents get larger income in period t

$$(Y_{2,t}, Y_{2,t+1}) = (2, 1)$$

4. Let's see how this affects the equilibrium and the well-being of both types.

Equilibrium: aggregate demand and supply

1. The consumption functions will come out as

$$C_{1,t} = \frac{1}{1+\beta}\left(1+\frac{1}{1+r_t}\right)$$
$$C_{2,t} = \frac{1}{1+\beta}\left(2+\frac{1}{1+r_t}\right)$$

2. Aggregate demand will be

$$Y_t^d = \frac{N}{1+\beta}\left(3+\frac{2}{1+r_t}\right)$$

3. Total supply in this economy is

$$Y_t^s = 3N$$

Equilibrium: interest rate and consumption

1. Equate demand with supply, and solve for r_t:

$$r_t = \frac{2}{3}\left(\frac{1}{\beta} - 1\right)$$

2. Note that this interest rate is smaller than it was when each type had equal endowments.
3. Plug in this interest rate to solve for the consumption of each type:

$$C_{1,t} = \frac{1 + 1.5\beta}{1 + \beta}$$
$$C_{2,t} = \frac{2 + 1.5\beta}{1 + \beta}$$

4. To solve for $t+1$ consumption, just note that from the Euler equation we have

$$C_{i,t+1} = \beta(1 + r_t)C_{i,t}$$

Equilibrium: aggregate saving

1. We can also look at the saving/borrowing behavior of both types.
2. For type 1 agents, we have:

$$\begin{aligned} S_{1,t} &= Y_{1,t} - C_{1,t} \\ &= 1 - \frac{1 + 1.5\beta}{1+\beta} = -\frac{0.5\beta}{1+\beta} \end{aligned}$$

3. For type 2 agents, we have

$$\begin{aligned} S_{2,t} &= Y_{2,t} - C_{2,t} \\ &= 2 - \frac{2 + 1.5\beta}{1+\beta} = \frac{0.5\beta}{1+\beta} \end{aligned}$$

4. Obviously, with $N_1 = N_2 = N$, aggregate savings are equal to zero

$$N_1 \cdot S_{1,t} + N_2 \cdot S_{2,t} = 0$$

Equilibrium: welfare

1. Let's see how well both types of agents are. Remember that $\beta = 0.9$
2. Consumption

$$\begin{aligned} C_{1,t} &= 1.2368 \quad , \quad C_{1,t+1} = 0.8246 \\ C_{2,t} &= 1.7632 \quad , \quad C_{2,t+1} = 1.1754 \end{aligned}$$

3. Utility:

$$\begin{aligned} U_{1,t} &= \ln(1.2368) + 0.9\ln(0.8246) = 0.039 \\ U_{2,t} &= \ln(1.7632) + 0.9\ln(1.1754) = 1.1754 \end{aligned}$$

4. Utility under "autarky"

$$\begin{aligned} U_{1,t} &= \ln(1) + 0.9\ln(1) = 0 \\ U_{2,t} &= \ln(2) + 0.9\ln(1) = 0.6931 \end{aligned}$$

5. It is easy to see which case is better

IV – Heterogeneity: case 2

— Permanently Rich Type 2 —

Permanently Rich Type 2

1. Type 1 households still have the same endowment pattern

$$(Y_{1,t}, Y_{1,t+1}) = (1, 1)$$

2. But the type 2 agents get larger income in **both periods**

$$(Y_{2,t}, Y_{2,t+1}) = (2, 2)$$

3. Guess what?
4. NO MORE GAINS FROM CONSUMPTION SMOOTHING.
5. WHY?

Equilibrium: aggregate demand and supply

1. The consumption functions will come out as

$$\begin{aligned} C_{1,t} &= \frac{1}{1+\beta}\left(1+\frac{1}{1+r_t}\right) \\ C_{2,t} &= \frac{1}{1+\beta}\left(2+\frac{2}{1+r_t}\right) \end{aligned}$$

2. Aggregate demand will be

$$Y_t^d = \frac{3N}{1+\beta}\left(1+\frac{1}{1+r_t}\right)$$

3. Total supply in this economy is

$$Y_t^s = 3N$$

Equilibrium: interest rate and consumption

1. Equate demand with supply, and solve for r_t:

$$r_t = \frac{1}{\beta} - 1$$

2. Note that this interest rate is smaller than it was when each type had equal endowments.
3. Plug in this interest rate to solve for the consumption of each type:

$$\begin{aligned} C_{1,t} &= \frac{1}{1+\beta}(1+\beta) = 1 \\ C_{2,t} &= \frac{2}{1+\beta}(1+\beta) = 2 \end{aligned}$$

4. So we are back to AUTARKY

Equilibrium: welfare

1. Consumption

$$\begin{aligned} C_{1,t} &= 1 \quad , \quad C_{1,t+1} = 1 \\ C_{2,t} &= 2 \quad , \quad C_{2,t+1} = 2 \end{aligned}$$

2. Utility:

$$\begin{aligned} U_{1,t} &= \ln(1) + 0.9 \ln(1) = 0 \\ U_{2,t} &= \ln(2) + 0.9 \ln(2) = 1.317 \end{aligned}$$

3. It is easy to see that the gains to Type 1 consumer from consumptions smoothing **have vanish**.
4. Consumer Type 2 gets the perfect consumtpion smoothing from autarky.
5. Therefore: **potential welfare gains from trade arise from differences, not similarities.**

V – The social planner case

What we did (and did not) in the previous section

1. In the previous section we saw three main things:
    1. Potential welfare gains from trade arise from differences, not similarities.
    2. Making someone richer temporarily, leads to a reduction in the interest rate
    3. Such reduction will lead to higher social welfare
2. It did not tell us that inequality ... was good. If inequality were good, making someone permanently richer, would increase social welfare even more. **That was not the case.**
3. Here, we will show that inequality, from a social welfare point of view, is bad.

The social planner

1. The social planner wants to maximize some weighted sum of the lifetime utility of both types of agents

$$W = \omega_1 N_1 (\ln C_{1,t} + \beta \ln C_{1,t+1}) + \omega_2 N_2 (\ln C_{2,t} + \beta \ln C_{2,t+1})$$

2. Faces the same resource constraint as the economy as a whole

$$\begin{aligned} \underbrace{N_1 \cdot C_{1,t} + N_2 \cdot C_{2,t}}_{Y_t^d} &= \underbrace{N_1 \cdot Y_{1,t} + N_2 \cdot Y_{2,t}}_{Y_t^s} \\ Y_{t+1}^d &= Y_{t+1}^s \end{aligned}$$

3. There is no trade between agents, and hence there are no prices in the social planner's problem.

The social planner's problem

1. The social planner wants to maximize some weighted sum of the lifetime utility of both types of agents

$$\max_{C_{1,t},C_{2,t},C_{1,t+1},C_{2,t+1}} W = \omega_1 N_1(\ln C_{1,t} + \beta \ln C_{1,t+1}) + \omega_2 N_2(\ln C_{2,t} + \beta \ln C_{2}$$

subject to

$$\begin{aligned} N_1 \cdot C_{1,t} + N_2 \cdot C_{2,t} &= N_1 \cdot Y_{1,t} + N_2 \cdot Y_{2,t} \\ N_1 \cdot C_{1,t+1} + N_2 \cdot C_{2,t+1} &= N_1 \cdot Y_{1,t+1} + N_2 \cdot Y_{2,t+1} \end{aligned}$$

2. To make things as easy as possible, assume

$$\begin{aligned} \omega_1 &= \omega_2 = \omega \\ N_1 &= N_2 = N \end{aligned}$$

3. Notice that N_1, N_2 **will vanish** from the constraints above

The Lagrangean function

1. The Lagrangean function looks like

$$\begin{aligned}\mathcal{L} &= \omega N(\ln C_{1,t} + \beta \ln C_{1,t+1}) + \omega N(\ln C_{2,t} + \beta \ln C_{2,t+1}) \\ &\quad + \lambda_t(Y_{1,t} + Y_{2,t} - C_{1,t} - C_{2,t}) + \lambda_{t+1}(Y_{1,t+1} + Y_{2,t+1} - C_{1,t+1} -\end{aligned}$$

2. First Order Conditions (FOCs) are

$$\begin{aligned}\partial\mathcal{L}/\partial C_{1,t} &= 0 \Rightarrow \frac{\omega N}{C_{1,t}} = \lambda_t \\ \partial\mathcal{L}/\partial C_{2,t} &= 0 \Rightarrow \frac{\omega N}{C_{2,t}} = \lambda_t \\ \partial\mathcal{L}/\partial C_{1,t+1} &= 0 \Rightarrow \frac{\omega N}{C_{1,t+1}} = \lambda_{t+1} \\ \partial\mathcal{L}/\partial C_{2,t+1} &= 0 \Rightarrow \frac{\omega N}{C_{2,t+1}} = \lambda_{t+1}\end{aligned}$$

Optimality form the FOCs

1. From the two first FOCs we get

$$C_{1,t} = C_{2,t}$$

2. And from the last two, we get

$$C_{1,t+1} = C_{2,t+1}$$

3. This means that the social planner would like to have perfect consumption equality.
4. Notice that this was obtained under the assumption of equal welfare weights $(\omega_1 = \omega_2 = \omega)$
5. What happens if one agent gets richer temporarily?

Temporarily Rich Type 2: Central Planner's consumption

1. Type 1 households still have the same endowment pattern

$$(Y_{1,t}, Y_{1,t+1}) = (1, 1)$$

2. But the type 2 agents get larger income in period t

$$(Y_{2,t}, Y_{2,t+1}) = (2, 1)$$

3. Total resources at t are equal to 3, so

$$C_{1,t} = C_{2,t} = 1.5$$

4. Total resources at $t+1$ are equal to 1, so

$$C_{1,t} = C_{2,t} = 1$$

Temporarily Rich Type 2: Central Planner's Social welfare

1. Utility is given by

$$\begin{aligned} U_{1,t} &= \ln(1.5) + 0.9\ln(1) = 0.4055 \\ U_{2,t} &= \ln(1.5) + 0.9\ln(1) = 0.4055 \end{aligned}$$

2. Social wellfare is given by

$$W = 0.4055 + 0.4055 = 0.811$$

3. Notice that under the decentralised outcome (or under the competitive outcome), social welfare were given by

$$W = 0.0390 + 0.7126 = 0.7516$$

4. It is easy to see that the Central Planners's solution is better
5. What should the Central Planner do: **tax the richer consumer by $T = 0.5$ and transfer this income to the poor consumer**

Why the Central Planner's solution is better

1. Individaul consumers like consumption smoothing ... **accross time**
2. The Central Planner likes the same ... but **accross different consumers**.
3. Notice two final things:
    1. All allocations above (all examples) are efficient
    2. But only one is optimal from the perspective of a social planner
4. Therefore:

Just because an allocation is efficient does not mean it is necessarily desirable from a social perspective.

VI – Bibliography

Bibliography

- For this particular topic read:

Eric Sims (2014). *"Intermediate Macroeconomics: Inequality"*, University of Notre Dame. Lecture Notes.

Read the entire paper.