ISCTE–IUL, Midterm Test, Modern Macroeconomics, BA in Economics, 16/04/2015.

ISCTE — INSTITUTO UNIVERSITÁRIO DE LISBOA

BA in Economics

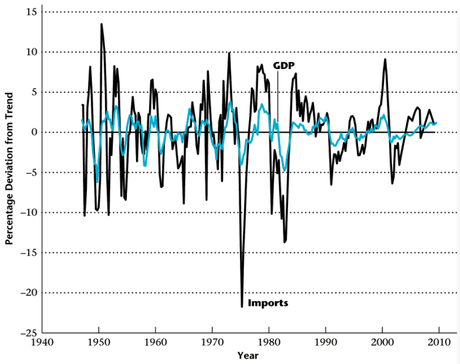
Modern Macroeconomics

Midterm test

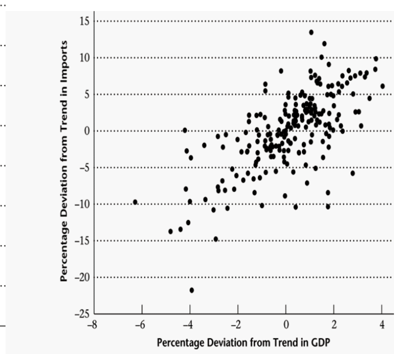
16 April– 2015 **Duration: 1h.30m**

Group I: Facts about business cycles (40 points)

1. Consider the next two figures. They represent the evolution of GDP and Imports with respect to the long term trend of each one, for the US economy in post World War II period.



Source: U.S. Department of Commerce, Bureau of Economic Analysis.



Source: U.S. Department of Commerce, Bureau of Economic Analysis.

As far as the main stylized facts of the business cycles are concerned, what do you conclude about the behavior of these two macroeconomic variables? **(20 points)**

2. One of the mostly used tool in modern macroeconomics is the Hodrick–Prescott filter (HP filter), which is given by the following expression

$$\min_{\tau_t} \sum_{t=1}^{T} \{(y_t - \tau_t)^2 + \lambda[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2\}$$

where y_t is the original series, τ_t is the smooth trend and $(y_t - \tau_t)$ is the HP filtered series. λ is a parameter. Answer the following two questions:

   (a) Why is it usually argued that the HP filter has significant advantages over the log linear filter? **(10 points)**

(b) What are the main criticisms directed at the HP filter. **(10 points)**

Answers Group I:

Question I-1. This question requires knowledge about the main concepts of short term business cycles: (i) countercyclical variables, pro-cyclical, and acyclical; (ii) leading, lagging and coincident variables, and (iii) volatility.

1. Is Imports procyclical, or ...? By just looking at the left panel it is difficult to have a clear answer about that. The right panel (a cross plot, not a time series) gives us some help: it looks very clearly procyclical, because correlation seems to be positive and relatively high.
2. Given the two panels above, it seems difficult to tell whether Imports is a leading, lagging or a coincident variable. We need some extra information to provide such an answer with some confidence in our arguments.
3. As far as volatility is concerned, it seems clear that Imports is much more volatile than real GDP. Moreover, the volatility of both variables declined significantly in the period post early 1980's, at least until the current crisis burst a few years ago.

Question I-2. The answer is expected to include:

1. The log linear filter suffers from two main criticisms:
   (a) Too much volatility
   (b) Recessions and booms with extremely large length
   (c) And If we correct those shortcomings by introducing "breacks", these are chosen arbitrarily and so ...
2. The main criticisms directed at the HP filter are:
   (a) Rewriting history: when the future shows itself, the past has to be rewritten, which is awkward.
   (b) Expansions and contractions are symmetric: reality violates this
   (c) A recession caused by a big contraction in aggregate demand will lead to a reduction in productive capacity: it does not make sense
   (d) Supply shocks are supposed to entirely explain what happens to the trend: if so, what kind of negative shocks have regularly caused the destruction of productive capacity?

Group II: Matlab (50 points)

1. Write a Matlab routine code in order to have in the same figure the following three functions represented:

$$\begin{aligned} y1 &= x^{0.5} \\ y2 &= x \\ y3 &= x^2 \end{aligned}$$

   with x defined in the interval $[0, 1]$. **(15 points)**

2. Write a Matlab routine code in order to have the following function represented in a figure:

$$x_{t+1} = 20 - 0.5x_t + \varepsilon_t$$

   where ε_t is a random variable, with mean equal to zero and variance equal to 1, $\varepsilon_t \tilde{} N(0, 1)$. In Matlab this random variable is written as: `randn(1)`. Simulate the dynamics of this process for $t = [1, 100]$. **(15 points)**

3. Suppose we have a file called *Data.txt*, which includes quarterly observations of the following three major macroeconomic variables: GDP (in column one), Money Supply (column 2), and the Inflation rate (column 3). The first observation is the first quarter of 2000, and the last one is the second quarter of 2014. Write a routine that graphically represents three panels in just one figure, each panel including the following: **(20 points)**

   (a) Panel one should include the time series of GDP and Money supply

   (b) Panel three should be a **yy** plot involving Money supply and the Inflation Rate.

   (c) Panel two should include a crossplot of GDP and the Money supply.

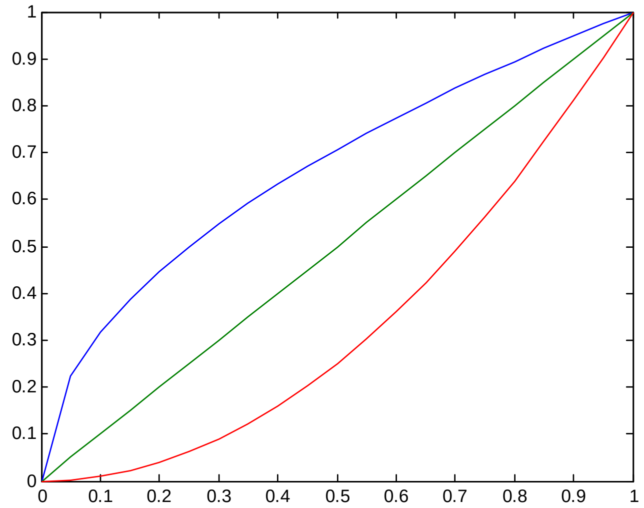
Answers Group II:

II-1. The following code will do the job:

```
x=0:0.05:1;
y1=x.^0.5;
y2=x;
y3=x.^2
plot(x,y1,x,y2,x,y3)
```

Figure 1 shows the functions $y1, y2, y3$ **(obviously, not required in the answer).**

II-2. The following code will do the job:




```
time=[0:100];
 x(1) = 10;
 for t=1:100;
 x(t+1)=20-0.5*x(t)+randn(1);
 end
 figure
 plot(time,x);
```

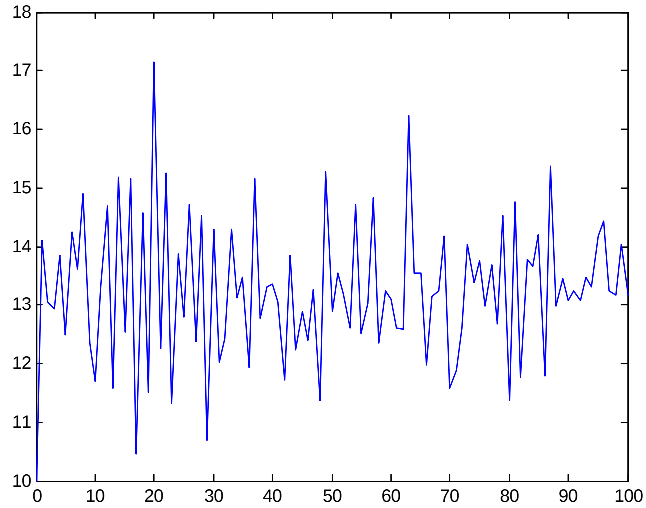
Figure 2 presentes the time series of the stochastic difference equation **(obviously, not required in the answer)**

II-3. The following code will do the job:

```
load Data.txt;
GDP=Data(:,1);
MS=Data(:,2);
Inf=Data(:,3);
time=2000.0:0.25:2014.25;
figure
subplot(121);plot(time,GDP,time,MS);
subplot(122);plotyy(time,MS,time,Inf);
subplot(123);plot(GDP,MS);
```

Notice that the following would also be a correct answer:

```
subplot(211);plot(time,GDP,time,MS);
subplot(212);plotyy(time,MS,time,Inf);
```




```
subplot(213);plot(GDP,MS);
```

Group III: Introduction to dynamics (40 points)

Consider an economic process that can be described by the following system

$$x_{t+1} = 0.5x_t + 2y_t$$
$$y_{t+1} = 5 + 0.2y_t$$

1. Does this system has a steady state (or a fixed point)? Explain. **(10 points)**
2. If there is a fixed point, is this unique or are there multiple equilibria? **(10 points)**
3. What is the type of stability in this process? Justify your answer. **(10 points)**
4. Assume now that the second equation is written as

$$y_{t+1} = 5 + 0.2y_t + \varepsilon_t$$

where ε_t is an external shock normally described as "white noise": $\varepsilon_t\tilde{}N(0,1)$. Does this shock produce any relevant changes to the previous results. Explain. **(10 points)**

Answers Group III.

III-1. Fisrtly, looking for a fixed point (a steady state) we should apply the condition:

$$x_{t+1} = x_t = \bar{x}$$
$$y_{t+1} = y_t = \bar{y}$$

Therefore, applying these two conditions to the system above we obtain

$$\begin{aligned}\bar{x} &= 0.5\bar{x} + 2(\bar{y}) \\ \bar{y} &= 5 + 0.2\bar{y}\end{aligned}$$

The solution is very easy to obtain:$\{[\bar{x} = 25, \bar{y} = 6.25]\}$

III-2. There is only one pair of $(\bar{x}, \bar{y})$, therefore there is a unique fixed point for each process, and no multiple equilibria.

III-3. The type of stability depends upon the values of the two parameters above (0.5 and 0.2). As each one of them is lower than 1 in modulus, each one of the two processes is stable.

III-4. No, as far as the expected values of of $(\bar{x}, \bar{y})$ are concerned. These will remain equal to $[E(\bar{x}) = 25, E(\bar{y}) = 6.25]$.

Group IV: Intertemporal decision making (70 points)

Assume that the utility of a representative consumer is a function of the level of consumption (c_t)

$$u(c_t) = 2\ln c_t.$$

Her/his objective is to maximize intertemporal utility discounted by a factor β

$$\max u(c_t) + \beta \cdot u(c_{t+1})$$

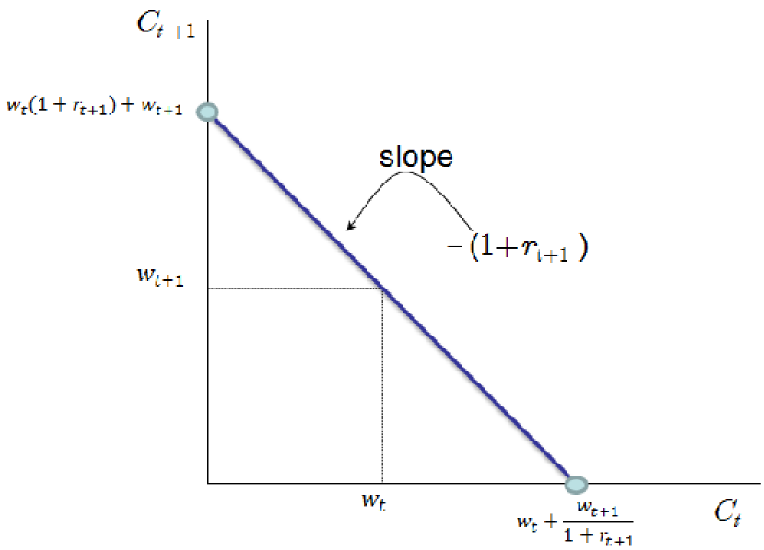
subject to the two usual constraints:

$$\begin{aligned}c_t + a_{t+1} &= w_t \\ c_{t+1} &= (1 + r_{t+1})a_{t+1} + w_{t+1}\end{aligned}$$

1. Derive and represent graphically the consolidated intertemporal constraint. **(10 points)**
2. Derive the Euler equation associated with this type of utility function: **(20 points)**

$$c_{t+1} = \beta(1 + r_{t+1})c_t$$

3. Determine the optimal consumption levels for each period, as well as the optimal savings level, by considering the following parameter values: $w_t = 100, w_{t+1} = 150, r_{t+1} = 5\%$, and $\beta = 0.9$. **(20 points)**
4. Now take into account that this consumer is not allowed to borrow more that 10% of his/her wages at t. Do you consider this consumer to be financially constrained? Explain graphically. **(10 points)**
5. In terms of social welfare, which situation is better: the initial situation or the new one with the financial constraint? Explain. **(10 points)**



Answers Group IV.

IV-1. Cancelling out a_{t+1} in the two constraints above leads to

$$c_t + \frac{c_{t+1}}{1+r_{t+1}} = w_t + \frac{w_{t+1}}{1+r_{t+1}} \tag{1}$$

see **Figure 3** for a graphical representation of this constraint.

IV-2. The derivation could be done in two different ways: by using the Lagrangian function, or by using the graphical analysis.

Let's consider the latter case. The slope of the intertemporal budget constraint is given by

$$-(1+r_{t+1})$$

The Marginal Rate of Substitution is given by

$$MRS_{t,t+1} = -\frac{\partial U/\partial c_t}{\partial U/\partial c_{t+1}} = -\frac{u'(c_t)}{\beta \cdot u'(c_{t+1})}$$

Equalizing both, we get

$$\begin{aligned} -\frac{u'(c_t)}{\beta \cdot u'(c_{t+1})} &= -(1+r_{t+1}) \\ u'(c_t) &= (1+r_{t+1}) \cdot \beta \cdot u'(c_{t+1}) \end{aligned}$$

Now, as $u'(c_t) = 2/c_t$ and $u'(c_{t+1}) = 2/c_{t+1}$,then,

$$c_{t+1} = \beta(1+r_{t+1})c_t.$$

IV-3. The Euler equation is given by

$$c_{t+1} = \beta(1 + r_{t+1})c_t$$

such that the Euler equation can be written as (notice that $\beta = 0.9$, and $r_{t+1} = 5\%$)

$$c_{t+1} = 0.9(1.05)c_t. \tag{2}$$

Now using eq. 1 and 2 we have a system of 2×2 (notice that $w_t = 100$, $w_{t+1} = 150$)

$$\begin{aligned} c_t + \frac{c_{t+1}}{1.05} &= 100 + \frac{150}{1.05} \\ c_{t+1} &= 0.9(1.05)c_t \end{aligned}$$

solution is: $\{[c_t = 127.82, c_{t+1} = 120.79]\}$.

IV-4. This consumer is a borrower. In period t, he/she receives an income of 100, but consumes 127.82, so she/he has to borrow around 27.8% of his/her income at t. Therefore, our consumer is financially constrained

$$27.8\% > 10\%$$

See **Figure 4** for graphical details.

IV-5. As the constraint is binding, the imposition of such a financial constraint will affect the wellfare of this consumer. The consumer would have a higher level of wellfare without the financial constraint. See **Figure 4** for graphical details. Without the constraint, the consumer would be in a higher indifference curve.

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