Intermediate Macroeconomics:
Inequality

Eric Sims
University of Notre Dame

Fall 2014

1 Introduction

This set of notes extends our basic equilibrium setup to include some heterogeneity among agents. I model this heterogeneity in terms of the endowments that agents receive.

Including this kind of heterogeneity allows to examine some interesting questions (albeit in an extremely stylized setting). These include:

- How is the well-being of an agent affected by another kind of agent being richer (having a bigger endowment)?

- How would a social planner (e.g. a government) reallocate resources in the face of heterogeneity in order to maximize social welfare?

The answers to these questions shed light on interesting topics relating to inequality and income redistribution. For the former, it turns out that any agent is (weakly) better off if other agents are richer. This is basic comparative advantage and trade at work. Gains from trade arise from differences, not similarities. In a sense, this suggests that inequality is good – a more unequal distribution of endowments actually makes people on the bottom end of the distribution better off. The mechanism through which this occurs is general equilibrium adjustment in prices – in our simple example, agents are who relatively poor may benefit from other agents being temporarily rich because this pushes down the equilibrium real interest rate, making it less costly for the poorer agents to borrow. But that's not the whole story. It turns out that a social planner would prefer a smooth allocation of consumption across types of agents. In an extreme case with a utilitarian social welfare function, a social planner would prefer perfect consumption equality. The reasons why a social planner would prefer to smooth consumption across agents are essentially the same as the reasons why an individual agent would prefer to smooth consumption across time. This calls for income redistribution among agents. Although we will not do it here, this desire to redistribute can be mapped into a more complicated world where there exist efficiency/equity tradeoffs. My objective here is to not draw any policy conclusions, but rather to give you a sense of the tradeoffs at play.

2 The Environment

There are two types of agents, 1 and 2. There are N_1 of type 1 agents and N_2 of type 2 agents. N_1 and N_2 are both sufficiently large that everyone behaves as price-takers. The agents are identical in terms of preferences, but potentially differ in terms of their endowments. In particular, agent 1 has known endowment stream $Y_{1,t}$ and $Y_{1,t+1}$, while agent 2 has known endowment stream $Y_{2,t}$ and $Y_{2,t+1}$. Either type of agent can borrow or save at real interest rate r_t.

Assume that the utility function is the natural log for both agents. The problem of an agent of type i ($i = 1$ or $i = 2$) is:

$$\max_{C_{i,t}, C_{i,t+1}} \ln C_{i,t} + \beta \ln C_{i,t+1}$$

$$\text{s.t.}$$

$$C_{i,t} + \frac{C_{i,t+1}}{1 + r_t} = Y_{i,t} + \frac{Y_{i,t+1}}{1 + r_t}$$

We can find the Euler equation as before. For agents of either type, we must have:

$$\frac{C_{i,t+1}}{C_{i,t}} = \beta(1 + r_t)$$

Since both agents have the same β and face the same interest rate, this tells us that the gross growth rate of consumption (the ratio of consumption tomorrow to consumption to today) will be the same for both type $i = 1$ agents and type $i = 2$ agents, though the levels of consumption need not be the same.

Solving for $C_{i,t+1}$ in terms of $C_{i,t}$ and plugging into the budget constraint, we can derive the consumption function:

$$C_{i,t} = \frac{1}{1 + \beta} \left(Y_{i,t} + \frac{Y_{i,t+1}}{1 + r_t} \right)$$

Note that this consumption function is the same for both types, though the levels of consumption will differ to the extent to which the endowments differ.

Total demand in this economy is the sum of consumption demand for each type. Since they are identical, all households of a given type will make identical consumption decisions. Therefore, total consumption for an agent of type i is $N_i C_{i,t}$, $i = 1$ or $i = 2$:

$$Y_t^d = N_1 C_{1,t} + N_2 C_{2,t}$$

Total supply is the sum of the endowments, where $N_i Y_{i,t}$ is the total endowment from agents of type i, $i = 1$ or $i = 2$:

$$Y_t^s = N_1 Y_{1,t} + N_2 Y_{2,t}$$

In equilibrium, the real interest rate, r_t, will adjust so that $Y_t^d = Y_t^s$, given the optimal consumption functions for each type given above.

In a general setting without the means to transfer resources across time through capital accumulation, one can think about general equilibrium working out so that *aggregate saving* is zero. What we did before with one type of agent is just a special case of this – there, we saw that saving of the agent would be zero, which equivalently meant that total expenditure had to equal the total endowment. The same thing is at work here, though individual saving need not be zero. Another way to express the market-clearing condition is that saving of one type of agent must equal borrowing of the other, so that there is no net borrowing or saving in the aggregate. Letting $S_{i,t} = Y_{i,t} - C_{i,t}$ be the saving of agent i ($i = 1$ or $i = 2$), we have:

$$N_1 S_{1,t} = -N_2 S_{2,t}$$

Plugging in the definition of saving, this becomes:

$$N_1 \left(Y_{1,t} - C_{1,t} \right) = N_2 \left(Y_{2,t} - C_{2,t} \right)$$

Re-arranging just gives us the condition that total "demand" (sum of consumption) must equal total "supply" (sum of endowments):

$$N_1 Y_{1,t} + N_2 Y_{2,t} = N_1 C_{1,t} + N_2 C_{2,t}$$

3   Some Examples

Now, let's use actual numbers for endowment streams to solve for the equilibrium interest rate under different scenarios. We also want to evaluate welfare, where I use the term welfare to denote the present discounted value of flow utility of an agent (e.g. $U_{i,t} = \ln C_{i,t} + \beta \ln C_{i,t+1}$ is welfare for agent i). We will also use the term welfare when talking about "social welfare," which is some function of the welfare of each type of agent. We want to look at how individual and social welfare are affected by different patterns of endowments, and examine how (if at all) a social planner could improve social welfare by eliminating endowment inequality.

Because it ends up being fairly innocuous, let's assume that there are an equal number of each type of agent in the economy: $N_1 = N_2 = N$. I will make use of this assumption for the remainder of these notes, which will simplify matters a good deal.

3.1   No Heterogeneity

To begin, let's assume that both types have the same endowment pattern of one unit in each period: $(Y_{1,t}, Y_{1,t+1}) = (1, 1)$ and $(Y_{2,t}, Y_{2,t+1}) = (1, 1)$. Given this endowment pattern, the consumption demand of each type is:

$$C_{1,t} = \frac{1}{1 + \beta} \left(1 + \frac{1}{1 + r_t} \right)$$

$$C_{2,t} = \frac{1}{1+\beta}\left(1 + \frac{1}{1+r_t}\right)$$

Total demand in the economy is then:

$$Y_t^d = 2N\left(\frac{1}{1+\beta}\left(1 + \frac{1}{1+r_t}\right)\right)$$

Total supply in this economy is:

$$Y_t^s = 2N$$

Equate demand and supply:

$$2N = \frac{2N}{1+\beta}\left(1 + \frac{1}{1+r_t}\right)$$

Solve for r_t:

$$1 + \beta = 1 + \frac{1}{1+r_t}$$
$$1 + r_t = \frac{1}{\beta}$$
$$r_t = \frac{1}{\beta} - 1$$

Now, plug this in to the consumption functions:

$$C_{1,t} = \frac{1}{1+\beta}(1+\beta) = 1$$

$$C_{2,t} = \frac{1}{1+\beta}(1+\beta) = 1$$

In other words, each household ends up consuming their endowment each period. Since consumption equals income for each type, there is no saving for either type. In other words, there is no trade between types. Plugging these first period consumption values into the Euler equation, one can show that consumption in the second period will also equal the endowment in that period: $C_{1,t+1} = C_{2,t+1} = 1$. This means that lifetime utility for either type is $U = \ln(1) + \beta\ln(1) = 0$. A quick reminder that utility is an *ordinal* concept – utility of 0 doesn't mean zero satisfaction.

3.2 Temporarily Rich Type 2

Now, let's change the setup in the following way. Type 1 households still have an endowment patten of $(Y_{1,t}, Y_{1,t+1}) = (1,1)$, but let's make the type 2 agents "rich" in the first period, with endowment pattern of $(Y_{2,t}, Y_{2,t+1}) = (2,1)$. Let's see how this affects the equilibrium and the well-being of both types.

The optimal consumption demands for both types are:

$$C_{1,t} = \frac{1}{1+\beta}\left(1 + \frac{1}{1+r_t}\right)$$

$$C_{2,t} = \frac{1}{1+\beta}\left(2 + \frac{1}{1+r_t}\right)$$

Total demand is:

$$Y_t^d = N\left(\frac{1}{1+\beta}\left(1 + \frac{1}{1+r_t}\right) + \frac{1}{1+\beta}\left(2 + \frac{1}{1+r_t}\right)\right)$$

$$Y_t^d = \frac{N}{1+\beta}\left(3 + \frac{2}{1+r_t}\right)$$

Total supply is:

$$Y_t^s = N + 2N = 3N$$

Equate demand with supply, and solve for r_t:

$$3N = \frac{N}{1+\beta}\left(3 + \frac{2}{1+r_t}\right)$$

$$3N = \frac{3N}{1+\beta}\left(1 + \frac{2/3}{1+r_t}\right)$$

$$1+\beta = 1 + \frac{2/3}{1+r_t}$$

$$\beta = \frac{2/3}{1+r_t}$$

$$\frac{3}{2}\beta = \frac{1}{1+r_t}$$

$$1 + r_t = \frac{2}{3}\frac{1}{\beta}$$

$$r_t = \frac{2}{3}\frac{1}{\beta} - 1$$

Note that this interest rate is smaller than it was when each type had equal endowments. Now let's plug in this interest rate to solve for the consumption of each type:

$$C_{1,t} = \frac{1}{1+\beta}\left(1 + \frac{3}{2}\beta\right) = \frac{1 + 1.5\beta}{1+\beta}$$

$$C_{2,t} = \frac{1}{1+\beta}\left(2 + \frac{3}{2}\beta\right) = \frac{2 + 1.5\beta}{1+\beta}$$

5

We can solve for consumption in the second period by noting that, for each type, we must have:

$$C_{i,t+1} = \beta(1 + r_t)C_{i,t}$$

Plugging in the expression for the interest rate, this becomes:

$$C_{i,t+1} = \frac{2}{3}C_{i,t}$$

This means that we'll have:

$$C_{1,t+1} = \frac{2}{3}\frac{1 + 1.5\beta}{1 + \beta}$$

$$C_{2,t+1} = \frac{2}{3}\frac{2 + 1.5\beta}{1 + \beta}$$

We can also look at the saving/borrowing behavior of both types. For type 1 agents, we have:

$$S_{1,t} = 1 - \frac{1 + 1.5\beta}{1 + \beta} = \frac{-0.5\beta}{1 + \beta}$$

$$S_{2,t} = 2 - \frac{2 + 1.5\beta}{1 + \beta} = \frac{2 + 2\beta - (2 + 1.5\beta)}{1 + \beta} = \frac{0.5\beta}{1 + \beta}$$

This here is a good opportunity to check that we haven't made a mistake. In aggregate, there can be no borrowing or saving in equilibrium – all output must be consumption. This means that the saving of each type should add up to zero:

$$N_1 S_{1,t} + N_2 S_{2,t} = 0$$

Using the assumption that $N_1 = N_2 = N$, we get:

$$N\left(\frac{-0.5\beta}{1 + \beta} + \frac{0.5}{1 + \beta}\right) = N \times 0 = 0$$

Hence, this all works out. The type 1 agents borrow (negative saving) while the type 2 agents save (positive saving). What drives this behavior is that type 2 agents want to smooth their consumption – they are natural first period savers, since they have more income in the first period than in the second. The fact that they want to save drives the real interest rate down (the equilibrium real interest rate here is lower than when both types of agents had equal endowments), which encourages the type 1 agents to borrow in the first period.

Now, let's see how well off both types of agents are. To do so, let's assume that $\beta = 0.9$. This means that consumption for each type will be:

$$C_{1,t} = 1.2368 \quad C_{1,t+1} = 0.8246$$
$$C_{2,t} = 1.7632 \quad C_{2,t+1} = 1.1754$$

In terms of lifetime utility, plugging these numbers in we have:

$$U_{1,t} = \ln(1.2368) + 0.9\ln(0.8246) = 0.0390$$
$$U_{2,t} = \ln(1.7632) + 0.9\ln(1.1754) = 0.7126$$

Recall, when both agents had an endowment of 1 in both periods, lifetime utility would be zero for both. In this example we've made type 2 "rich" by giving them more of an endowment in period 1. Naturally, they have higher lifetime utility. But interestingly, we also see that type 1 agents have higher lifetime utility. Put differently, even though they aren't any richer, type 1 agents are better off – they benefit from type 2 agents being richer. This economy now has more inequality relative to the earlier case, but both types of agents are better off, including the ones who haven't had any change in endowment.

It is useful to compare these numbers to what would happen under "autarky," which means that each agent just consumes his/her endowment each period. If that were the case, utility of each type would be:

$$U_{1,t} = \ln(1) + 0.9\ln(1) = 0$$
$$U_{2,t} = \ln(2) + 0.9\ln(1) = 0.6931$$

In other words, both types of agents would be worse off under autarky than they are by trading with each other. The existence of trade allows the rich type 1 agents to shift some of their consumption to the second period, which given their smoothing motive, makes them better off. The desire of type 2 agents to save in the first period drives down the equilibrium real interest rate, which allows type 1 agents to face a lower interest rate and increase their consumption in the first period. Both parties end up better off, though type 2 agents have a much larger increase in welfare than type 1 agents.

3.3 Permanently Rich Type 2

In the previous subsection, I made type 2 rich in the first period, but they had the same endowment as type 1 agents in the second period. Now, let's consider the case where the endowment stream of type 2 agents is $(Y_{2,t}, Y_{2,t+1}) = (2, 2)$. Everything else is the same.

With this endowment pattern, consumption demand for each type of agent is:

$$C_{1,t} = \frac{1}{1+\beta}\left(1 + \frac{1}{1+r_t}\right)$$

$$C_{2,t} = \frac{1}{1+\beta}\left(2 + \frac{2}{1+r_t}\right) = \frac{2}{1+\beta}\left(1 + \frac{1}{1+r_t}\right)$$

Total demand in this economy is:

$$Y_t^2 = \frac{N}{1+\beta}\left(1 + \frac{1}{1+r_t}\right) + \frac{2N}{1+\beta}\left(1 + \frac{1}{1+r_t}\right) = \frac{3N}{1+\beta}\left(1 + \frac{1}{1+r_t}\right)$$

Total supply is:

$$Y_t^s = N + 2N = 3N$$

Equating demand with supply, we have:

$$1 = \frac{1}{1+\beta}\left(1 + \frac{1}{1+r_t}\right)$$

Solve for the real interest rate:

$$1 + \beta = 1 + \frac{1}{1+r_t}$$

$$\frac{1}{1+r_t} = \beta$$

$$1 + r_t = \frac{1}{\beta}$$

$$r_t = \frac{1}{\beta} - 1$$

Note that this is the same equilibrium real interest rate as when type 2 had the same endowment pattern as type 1. Plug this interest rate in to the consumption functions to derive consumption in each period:

$$C_{1,t} = \frac{1}{1+\beta}(1+\beta) = 1$$

$$C_{2,t} = \frac{2}{1+\beta}(1+\beta) = 2$$

In other words, we see here that each type of agent just consumers her endowment. This means that neither agent saves or borrows – there is no trade between them. One can then show that each agent will also consumer her endowment in period $t+1$. Calculating lifetime utility for each type:

$$U_{1,t} = \ln(1) + \beta\ln(1) = 0$$

$$U_{2,t} = \ln(2) + \beta \ln(2) = 1.3170$$

We can see here that type 1 agents have the same overall level of welfare (lifetime utility) as in the baseline case where both types of agents have the same endowment stream. Naturally, type 2 people are better off, both relative to the case where they had an endowment pattern of $(1, 1)$ and relative to the case where they had $(2, 1)$. Type 2 being better off here has no effect on type 1 in equilibrium – they are no better off, but they are no worse off either.

3.4 What's Going on Here: Comparative and Absolute Advantage

What's going on here is nothing more than simple comparative and absolute advantage from principles of micro. Think of the endowment as a good, and think of each type of agent as countries. Importantly, here we think about the endowment in each period as a different kind of good – $Y_{i,t}$ is a good, and $Y_{i,t+1}$ is a different good, even though we conceptualize them as the same thing (e.g. fruit). What differentiates them is time.

We can think about each type as "producing" two goods – fruit today, and fruit tomorrow. I put "produce" in quotation marks because there is no production in a pure endowment economy. A type has an absolute advantage if it has a bigger endowment in either period. In subsection 3.1, neither type had an absolute advantage. In subsection 3.2, type 2 agents had an absolute advantage in period t fruit, but neither type had an absolute advantage in period $t + 1$ fruit. In subsection 3.3, type 2 had an absolute advantage in both period t and period $t + 1$ fruit.

What about comparative advantage? You can figure out comparative advantage by looking at the ratio of the first period endowment to the second period endowment – this will tell you which type is *relatively* better at "producing" the first period endowment. In subsection 3.1, these ratios were 1 for both types, so neither had a comparative advantage in first or second period fruit. In subsection 3.2, the ratio was 1 for type 1, but 2 for type 2. This means that, in that specification, type 2 agents hold a comparative advantage in period t fruit. In contrast, they have a comparative disadvantage in period $t + 1$ fruit (the ratio of their period $t + 1$ endowment to period t is $1/2$, less than for type 1 agents). In subsection 3.3, even though type 2 agents have an absolute advantage in both types of goods (period t and period $t + 1$), they have no comparative advantage in period t goods since the ratio of endowments across time is 1, the same as for the type 1 agent.

Basic trade theory tells us that countries/agents ought to "specialize" in that thing in which they have a comparative advantage, trading some of that away in exchange for the good in which they have a comparative disadvantage. In subsections 3.1 and 3.3, neither type of agent had a comparative advantage, so there was no trade between agents. In subsection 3.2, however, type 2 agents had a comparative advantage in period t fruit. What we saw in equilibrium there was that type 2 agents traded away some of their first period endowment (consumption of 1.76, but endowment of 2) in exchange for some more second period consumption (consumption of 1.17, but endowment of 1). Type 2 "specialized" in period t goods, "exporting" them away to type 1 agents. In exchange, type 2 agents got some extra period $t + 1$ consumption. And, as we saw above, *both*

types of agents were made better off relative to the case of no trade ("autarky").

A more general point here is that the ability to trade makes you no worse off, and may well make you better off. In the stylized example I've given, you can always consume your endowment and do no saving. This is equivalent to the autarky case, and so represents a worst case scenario of sorts. If another type becomes richer than you, you cannot be worse off because it's always possible to simply consume your endowment. But you could be better off.

Potential welfare gains from trade arise from *differences*, not similarities. This is a broad point in all of economics. If I make type 2 richer than type 1 for the first period, both types end up benefitting. Yes, the distribution of income is more unequal, but type 1 agents nevertheless end up with higher welfare. In fact, if you are a type 1 agents, you'd prefer the type 2 person to be even more different than you – in subsection 3.2, had I made the first period endowment of type 2 agents to be 3 instead of 2, type 1 agents would be even better off. Of course, type 1 agents would rather have a higher endowment themselves, but they nevertheless still benefit from the other type of agent being richer.

This all suggests that, in a sense, inequality is "good" (or at least not bad). If I make the agents more different from one another, they both benefit because gains from trade arise from differences, not similarities.

4   Social Welfare and a Planner's Problem

The analysis above suggests that inequality may be "good" in the sense that both types are better off when I make one type richer. In this subsection I explore the converse – that from the perspective of social welfare (the sum of welfare of each type), inequality is bad.

Let's consider the problem of a fictitious "social planner." The social planner wants to allocate first and second period consumption to both types of agents to maximize some weighted sum of the lifetime utility of both types of agent. She faces the same resource constraint as the economy as a whole above (total consumption cannot exceed the total endowment), but there is no trade between agents, and hence there are no prices in the social planner's problem.

The problem can be written:

$$\max_{C_{1,t},C_{1,t+1},C_{2,t},C_{2,t+1}} \mathcal{W} = \omega_1 N_1 \left(\ln C_{1,t} + \beta \ln C_{1,t+1} \right) + \omega_2 N_2 \left(\ln C_{2,t} + \beta \ln C_{2,t+1} \right)$$

s.t.

$$N_1 C_{1,t} + N_2 C_{2,t} = N_1 Y_{1,t} + N_2 Y_{2,t}$$

$$N_1 C_{1,t+1} + N_2 C_{2,t+1} = N_1 Y_{1,t+1} + N_2 Y_{2,t+1}$$

Here, ω_1 and ω_2 are weights placed on the lifetime utility of each type. Hence, social welfare, $\mathcal{W}$, is equal to ω_1 times total lifetime utility for type 1 agents (the number of agents, N_1, times the lifetime utility of type 1 agents) plus ω_2 times total lifetime utility for type 2 agents. The constraints simply say that total consumption must equal the total endowment in each period.

To make things simple and transparent, let's suppose that $\omega_1 = \omega_2 = \omega$ and that $N_1 = N_2 = N$. This meas that the N terms drop out of the constraints:

$$C_{1,t} + C_{2,t} = Y_{1,t} + Y_{2,t}$$

$$C_{1,t+1} + C_{2,t+1} = Y_{1,t+1} + Y_{2,t+1}$$

We can eliminate the constrains by solving for $C_{2,t}$ and $C_{2,t+1}$ in terms of type 1 consumptions and endowments, which allows us to write the problem as unconstrained:

$$\max_{C_{1,t},C_{1,t+1}} \mathcal{W} = \omega N \left(\ln C_{1,t} + \beta \ln C_{1,t+1} \right) + \omega N \left(\ln \left(Y_{1,t} + Y_{2,t} - C_{1,t} \right) + \beta \ln \left(Y_{1,t+1} + Y_{2,t+1} - C_{1,t+1} \right) \right)$$

Take the derivatives with respect to $C_{1,t}$ and $C_{1,t+1}$:

$$\frac{\partial \mathcal{W}}{\partial C_{1,t}} = \omega N \frac{1}{C_{1,t}} - \omega N \frac{1}{Y_{1,t} + Y_{2,t} - C_{1,t}}$$

$$\frac{\partial \mathcal{W}}{\partial C_{1,t+1}} = \omega N \beta \frac{1}{C_{1,t+1}} - \omega N \beta \frac{1}{Y_{1,t+1} + Y_{2,t+1} - C_{1,t+1}}$$

Note that $Y_{1,t} + Y_{2,t} - C_{1,t} = C_{2,t}$ and $Y_{1,t+1} + Y_{2,t+1} - C_{1,t+1} = C_{2,t+1}$ from the budget constraints. Using this, and setting the derivatives equal to zero, we get:

$$\frac{1}{C_{1,t}} = \frac{1}{C_{2,t}}$$

$$\frac{1}{C_{1,t+1}} = \frac{1}{C_{2,t+1}}$$

In other words, the social planner would like to equate the marginal utilities of consumption for both types in both periods. But this means that the social planner would like to equalize consumption of each type in each period:

$$C_{1,t} = C_{2,t}$$

$$C_{1,t+1} = C_{2,t+1}$$

This means that the social planner (subject to the caveat here that I have assigned equal welfare weights) would like to have perfect consumption equality.

In the case from Section 3.1 where the endowment streams of both types of agents were the same, the social planner would choose the same consumption bundles that are chosen by the agents. This will not be the case in the situations where type 2 agents are richer, however. Consider the case where type 2 agents are rich in the first period. Perfect consumption equality would mean that in the first period each type of agent gets 1.5 units of consumption and in the second period each type gets 1 unit. In the first period this requires redistribution away from type 2 agents to

type 1 agents (the planner takes 0.5 units away from type 2, and gives it to type 1). Let's look at utility of each type under this redistribution.

$$U_{1,t} = \ln(1.5) + 0.9\ln(1) = 0.4055$$
$$U_{2,t} = \ln(1.5) + 0.9\ln(1) = 0.4055$$

If we sum up these utilities (e.g. here I set $\omega = 1$), we get:

$$\mathcal{W} = 0.4055 + 0.4055 = 0.8110$$

This overall level of social welfare can be seen to be higher than under the competitive arrangement where the agents trade with one another:

$$\mathcal{W} = 0.0390 + 0.7126 = 0.7516$$

Here we see that a social planner, assuming she puts equal weight on welfare of both types of agents (what is sometimes called a "utilitarian social welfare function"), would prefer to equalize consumption across agents. To the extent to which the endowments are different, this requires taxing away endowment from the rich agents and redistributing it to the relatively poor agents.

In practice, how would a social planner implement its desired equilibrium? It would have to do so through taxes. In particular, suppose that agents have to pay a tax each period, and allow the tax to differ by type. This means that "net income" for an agent is $Y_{i,t} - T_{i,t}$ in period t, and $Y_{i,t+1} - T_{i,t+1}$ in period $t + 1$, for agents of types $i = 1$ and $i = 2$. These taxes can be negative, in which case we would call them a transfer. What the planner would do is work backwards. She would figure out the desired allocation of consumption among agents by solving the social planner's problem. Then she would pick $T_{1,t}$, $T_{2,t}$, $T_{1,t+1}$, and $T_{2,t+1}$ to implement this as an equilibrium outcome, subject to the restriction that $T_{1,t} + T_{2,t} = 0$ and $T_{1,t+1} + T_{2,t+1} = 0$ (these restrictions imply that there is no waste).

Let's focus on the case where type 2 agents are rich in the first period (Section 3.2). We saw there that the equilibrium outcome was for $C_{1,t} = 1.2368$ and $C_{2,t} = 1.7632$. From above, we know that the planner would like to implement an outcome in which $C_{1,t} = C_{2,t} = 1.5$. It is straightforward to show that this can be accomplished by setting $T_{2,t} = 0.5$ and $T_{1,t} = -0.5$: in other words, the planner taxes the rich agent 0.5 units of the endowment and redistributes it to the poor agent. With those new endowments, the equilibrium allocation will align with the planner's solution. So the planner's solution can be implemented with redistribution through taxes. Note that these are relatively special kinds of taxes, though – the amount an agent has to pay (or receives) doesn't depend on how much he/she works or saves. We sometimes call such taxes "lump sum" taxes – every agent of a particular type has to pay a fixed (i.e. lump) sum to the government.

The intuition for why the planner would prefer perfect consumption equality across agents is essentially the same as why an individual agent would prefer to smooth consumption across time. We

12

have assumed that the utility function is concave, which means that there is diminishing marginal utility. If you have two agents with identical utility functions, one with low current consumption and one with high consumption, you can increase the *sum* of utilities by taking consumption away from the agent with high consumption and giving it to the agent with low consumption. This is because the agent with low consumption has a high marginal utility of consumption and the agent with high consumption has a low marginal utility. Given these differences in marginal utilities, a social planner lowers the rich agent's utility by less than she increases the utility of the poor agent by redistributing the endowment. This involves making the rich agent worse off, but this is made up for by making the poorer agent better off. These conclusions would not necessarily follow if the welfare weights the planner uses were different from one another – but as long as the planner places some weight on the less well-off agents, there will be some redistribution.

4.1   Pareto Efficiency and Welfare

Economists like to talk about "efficiency." What do we mean by that word?

A widely accepted concept of efficiency is that of *Pareto efficiency*. An allocation of resources is Pareto efficient if it is not possible to increase the welfare of one agent without harming someone else. In all the examples I gave above, the allocations are all Pareto efficient. In an endowment setting, the only way an allocation could be Pareto inefficient is if there were unused resources – e.g. if the total endowment were 2, agent 1 consumed 0.95, and agent 2 consumed 1; you could increase the welfare of agent 1 by increasing her consumption without harming agent 2. But when the full endowment is consumed, it is not possible for the allocation to be inefficient. Pareto inefficiency is more likely to emerge in economies with production. There, things like monopoly power or externalities can yield Pareto inefficiencies.

An efficient allocation does *not* necessarily imply a desirable allocation from the perspective of social welfare. We can see this at play above. The equilibrium allocations are all Pareto efficient, but the social planner would prefer a different allocation in the case where there are endowment differences in the first period. In other words, just because an allocation is efficient does not mean it is necessarily desirable from a social perspective. The converse is different, at least in an endowment economy with no frictions impeding the transfer of resources among agents: a desirable allocation cannot be inefficient, because some people could be made better off, and no one left worse off, by moving to an efficient allocation.

5   Efficiency/Equity Tradeoffs and the Real World

The example economy I've described in these notes is highly stylized: there is no production, and implicitly the government has at its disposal the authority/ability to transfer resources among agents. While this is a good, clean framework for making several points, it's not the way the real world works.

In the real world, economies aren't endowed with output, output has to be produced. Output

is produced using labor and capital, and labor and capital are paid for their services in this production. Governments also typically don't have access to simple transfer schemes between households. Rather, they use tax rates on things like labor and capital income.

Issues of inequality and redistribution are a lot more complicated in a production structure were transfers of resources are achieved through tax rates on capital and labor income. For the reasons laid out above, it is still the case that a social planner with a utilitarian welfare function would like to equalize consumption across households. This argues for eliminating inequality. But to the extent to which this transfer of resources has to come through tax rates on capital and labor, there exists a tradeoff. Taxing owners of capital and suppliers of labor reduces their incentives to work and invest – one can show that positive tax rates on these sources of income results in an inefficient allocation (in essence, there is too little overall production). So, a government faces a non-trivial problem in such an economy. On the one hand, it may like to redistribute resources away from the well-off and toward the less well-off. On the other hand, doing so may require taxation that leads to inefficiencies and lowers the overall "size of the pie." How much redistribution you'd want to do in such a world depends on several factors: how strong the desire to redistribute is (which depends on the form of the social welfare function and the concavity of utility functions) and how much taxation of capital and labor reduces the size of the pie. The latter depends on how elastic labor and capital supply are. These are empirical questions, and there are strong differences of opinion concerning how much inefficiency taxation creates. As I said in the Introduction, my objective here is not to supply you with answers, but rather to expose you to the relevant tradeoffs at play.