Overlapping Generations & Social Security Systems

— Week 7 —

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Summary

1. Why overlapping generations?
2. Controversies
3. The OLG model
4. Optimal allocations
5. Decentralized or competitive allocations
6. An example
7. PAYG versus Fully Funded social security systems

I – Why Overlapping Generations?

The nature of the problem

1. Markets are **incomplete:** economic agents are finite-lived, and agents currently alive cannot trade with the unborn.
2. As a result:
    1. Competitive equilibria need not be **Pareto optimal**
    2. Ricardian equivalence **does not hold**.
3. Thus, the timing of taxes and the size of the government debt matters.
4. Without **government intervention**, resources may not be allocated optimally among generations, and capital accumulation may be suboptimal.
5. **Redistributing wealth** among generations may increase social welfare.

The two social security systems

1. Overlapping generations are very useful to explain social security systems
2. There are two types of social security systems:
    1. "Pay-As-You-Go" system
    2. "Fully Funded" system
3. The system that today is largely dominant is: PAYG
4. "We do not believe the currently projected long run growth rates of Social Security and Medicare are sustainable under current financing arrangements" (Hakkio and Wiseman, page 1)
5. Why?

The two social security systems (cont.)

1. The reason is associated with the following result:

$$(1+g)(1+n) < (1+r)$$

   Growth rates of: $n-$population, $g-$ income, $r-$ financial return

2. We will show that the previous condition implies the bankruptcy of **currently PAYG** systems over the long term
3. Huge technical problems to change the current PAYG system:
   1. **What to do with the last generation?**
   2. How to do it? We live in open and democratic societies, so ...
4. There is a **terrible controversy** about this issue

II – Controversies

Edward Prescott and the current situation

> *"**Myth No. 5**: Government debt is a burden on our grandchildren [...] Theory and practice tell us that the optimal amount of public debt that maximizes the welfare of new generations of entrants into the workforce is two times gross national income, or GDP. This assumes 1% population growth, 2% productivity growth, 4% real after-tax return on investments, and that people work to age 63 and live to age 85. Currently, privately held public debt is about 0.3 times GDP, and if we include our Social Security obligations, it is 1.6 times GDP. In either case, we could argue that we have too little debt."*

Edward Prescott (Nobel in Economics, 2004) Wall Street Journal, 11 December 2006.

Robert Samuelson

> *"Would Franklin Roosevelt approve of Social Security? The question seems absurd. After all, Social Security is considered the New Deal's signature achievement. It distributes nearly $800 billion a year to 56 million retirees, survivors and disabled beneficiaries. On average, retired workers and spouses receive $1,839 a month — money vital to the well-being of millions. Roosevelt would surely be proud of this, and yet he might also have reservations. Social Security has evolved into something he never intended and actively opposed."*

Robert Samuelson, "Would Roosevelt recognize today's Social Security?", Washington Post, 9 April, 2012

Paul Krugman

"Social security medicare and medicaid Strikes Again

Jared Bernstein and Dean Baker are both mad, understandably, at Robert Samuelson, who pulls out, for the 7 millionth time, the old Social Security bait and switch. Here's how it works: to make the quite mild financial shortfall of Social Security seem apocalyptic, the writer starts out by talking about Social Security, then starts using numbers that combine SS with the health care programs — programs that are very different in conception, financing, and solutions.

And then the writer ends by demanding that we cut Social Security, as opposed to addressing health care costs."

Paul Krugman, New York Times, 9 April 2012

Dean Baker

> *"Today's column by Robert Samuelson tries to tell us that Franklin Roosevelt would be appalled by the current state of the Social Security program. Of course, he produces not a single iota of evidence to support this position, although it is very clear that Samuelson doesn't like Social Security."*

Dean Baker, "Robert Samuelson Shows that the Post Has no Fact Checkers on Its Opinion Pages", CEPR, London, Sunday, 08 April 2012 21:28.

Franklin Roosevelt

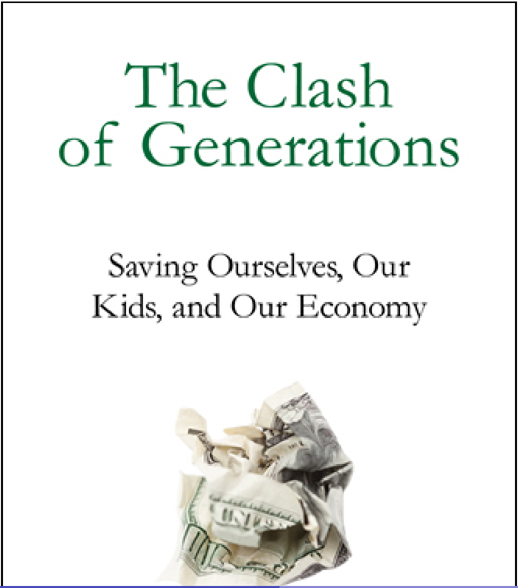
"This is the same old dole under another name. It is almost dishonest to build up an accumulated deficit for the Congress ... to meet."

"We put those payroll contributions there so as to give the contributors a legal, moral and political right to collect their pensions ... No damn politician can ever scrap my Social Security program."

Franklin Roosevelt (1935)

Cited in Robert Samuelson above, Washington Post, 9 April, 2012

Laurence J. Kotlikoff and Scott Burns

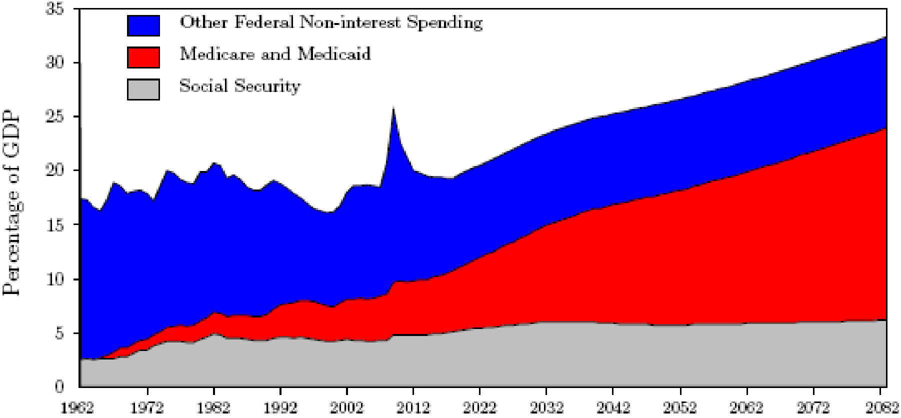


The United States is bankrupt, flat broke. Thanks to accounting that would make Enron blush, America's insolvency goes far beyond what our leaders are disclosing. The United States is a fiscal basket case, in worse shape than the notoriously bailed-out countries of Greece, Ireland, and others. How did this happen? In The Clash of Generations, experts Laurence Kotlikoff and Scott Burns document our six-decade, off-balance-sheet, unsustainable financing scheme. They explain how we have balanced our longer lives on the backs of our (relatively few) children. At the same time, we've been on a consumption

Simon Johnson and James Kwak in "White House Burning"

"In the 1950s, payments for individuals (such as social security checks or, today, Medicare reimbursements) made up barely one-fifth of federal spending. Today they account for three-fifths ... Cutting federal spending means taking money directly out of people's pockets ... the idea that we can balance the budget purely by eliminating unspecified 'bad' government programmes is a fantasy."

Dismiss at your peril



Source: CBO Long-Term Budget Outlook (June 2009)

III -The OLG model

Consumers

1. Time: $t = 1, 2, 3, ...$
2. In every period t, a new generation is born, which will live for two periods: t and $t + 1$.
3. This generation works and accumulates savings during t, retires in $t + 1$.
4. In $t + 1$, consumers use their savings invested at t to finance their consumption

Consumers (cont.)

1. Population (or the labor force) (L) grows at a constant annual rate (n)

$$L_{t+1} = (1+n)\ L_t \tag{1}$$

2. **Consumption** of an agent in each generation is c_t^y , c_{t+1}^o ("y" for "young", "o" for "old")

$$u(c_t^y, c_{t+1}^o)$$

3. **Boooo!** An elegant way of saying that utility is well behaved:

$$\begin{aligned} v(c_t^y, c_{t+1}^o) &= TMS_{t,t+1} = -\frac{\partial u/\partial c_t^y}{\partial u/\partial c_{t+1}^o} \\ & \quad \textit{then} \\ lim_{c^y \to 0} v(c_t^y, c_{t+1}^o) &= \infty \ , \ \ lim_{c^o \to 0} v(c_t^y, c_{t+1}^o) = 0 \end{aligned}$$

Firms

1. **Firms** produce output (Y_t) with capital (K_t) and labor (L_t)

$$Y_t = F(K_t, L_t)$$

2. The production function is linear homogeneous
3. Output can be consumed or invested
4. Firms maximize profits

Types of allocations

Pareto optimum

Pareto optimum is achieved when you can not improve the welfare of one agent without making some other agent worse off.

1. There are **two types** of economic allocations:
   1. Optimal allocations: those that satisfy the Pareto optimum
   2. Decentralized or competitive allocations: they may (or may not) satisfy the Pareto optimum
2. Next section we will discuss the optimal allocations
3. In section V we will deal with decentralized (competitive) allocations

IV - Optimal allocations

Optimal allocations

1. Those that are made by **a social planner**
2. Assume this planner has control over production, capital accumulation, and the distribution of consumption goods between the young and the old.
3. This planner has the power to correct the decisions by private agents if they do not guarantee the Pareto optimum
    1. Imposing taxes, granting subsidies, leading to a redistribution of income across generations if necessary
4. This social planner determines:
    1. The optimal levels of consumption per capita of both generations (c_t^o, c_t^y), in each period of time
    2. The optimal levels of savings per capita (s_t^*), and the capital stock per capita (k_{t+1}^*) in each period

Optimal allocations (cont.)

1. The central planner maximizes consumption for each generation (new and old) for the same period of time (same t), with $t = 0, 1, 2, 3, ..$
   1. maximization across **each column** (see table below)
2. In the competitive equilibrium, each generation maximizes its own consumption between t and $t+1$
   1. maximization across **each line** (see table below)

t	$t+1$	$t+2$	$t+3$	$t+4$
c^o_t				
c^y_t	c^o_{t+1}			
	c^y_{t+1}	c^o_{t+2}		
		c^y_{t+2}	c^o_{t+3}	
			c^y_{t+2}	...
				...

The social planner constraint

1. **The constraint** of the "social planner" is

$$F(K_t, L_t) = c_t^y \cdot L_t + c_t^o \cdot L_{t-1} + \underbrace{(K_{t+1} - K_t)}_{\Delta K}$$

2. Let's us transform this constraint in per capita values, dividing it through by L_t

$$\frac{F(K_t, L_t)}{L_t} + \frac{K_t}{L_t} = \frac{K_{t+1}}{L_t} + \frac{c_t^y \cdot L_t}{L_t} + \frac{c_t^o \cdot L_{t-1}}{L_t} \quad (2)$$

3. Defining the new variables in per capita values as

$$f(k_t) = \frac{F(K_t, L_t)}{L_t} \quad , \quad k_t = \frac{K_t}{L_t}$$

4. Equation (2) can be written as

$$f(k_t) + k_t = (1+n)k_{t+1} + c_t^y + \frac{c_t^o}{1+n} \quad (3)$$

Definition: Pareto optimal allocation

Definition

"A Pareto optimal allocation is a sequence $\{c_t^y, c_t^o, k_{t+1}\}_{t=0}^{\infty}$ *satisfying (3) and the property that there exists no other allocation* $\{\hat{c}_t^y, \hat{c}_t^o, \hat{k}_{t+1}\}_{t=0}^{\infty}$ *which satisfies (3) and*

$$\begin{aligned} \hat{c}_1^o &\geq c_1^o \\ u(\hat{c}_t^y, \hat{c}_{t+1}^o) &\geq u(c_t^y, c_{t+1}^o) \end{aligned}$$

for all $t = 0, 1, 2, 3, \ldots$*, with strict inequality in at least one instance."*

Examples: Pareto optimal allocations

1. Assume the following example of three different allocations of resources across generations: **A**, **B**, **C**.
2. Assume that the same amount of resources for the three examples:

$$TR = 8 \text{ euros}$$

3. Assume that the utility function is given by

$$u(c_t^y, c_{t+1}^o) = c_t^y + c_{t+1}^o$$

4. Which allocation is optimal from a Pareto point of view?

Allocation A

t	$t+1$	$t+2$	...
$c_t^o = 4$			
$c_t^y = 4$	$c_{t+1}^o = 4$		
	$c_{t+1}^y = 4$	$c_{t+2}^o = 4$	
		$c_{t+3}^o = 4$	...
			...
$TR = 8$	$RT = 8$	$RT = 8$	

Allocation B

t	$t+1$	$t+2$	...
$c_t^o = 3$			
$c_t^y = 5$	$c_{t+1}^o = 3$		
	$c_{t+1}^y = 5$	$c_{t+2}^o = 3$	
		$c_{t+3}^o = 5$	...
			...
$TR = 8$	$RT = 8$	$RT = 8$	

Allocation C

t	$t+1$	$t+2$	...
$c_t^o = 5$			
$c_t^y = 3$	$c_{t+1}^o = 5$		
	$c_{t+1}^y = 3$	$c_{t+2}^o = 5$	
		$c_{t+3}^o = 3$	...
			...
$TR = 8$	$RT = 8$	$RT = 8$	

Which allocation is superior?

1. Which allocation is optimal from a Pareto point of view?
2. The utility levels are all equal:

$$\begin{aligned} u(c_t^y, c_{t+1}^o)_C &= 5+3=8 \\ u(c_t^y, c_{t+1}^o)_A &= 4+4=8 \\ u(c_t^y, c_{t+1}^o)_B &= 3+5=8 \end{aligned}$$

3. However, the welfare of the first (or initial) old generation is

$$c_1^o(C) = 5 > c_1^o(A) = 4 > c_1^o(B) = 3.$$

4. Allocation C is superior to A, which in turn is superior to B.
5. How could a central planner transform allocations A and B into optimal?
6. Taxing the young generation (1 euro in A; 2 euros in B) and transferring that income to the old generation.

The optimal problem for the social planner

1. The optimal problem for the "social planner" is maximize

$$\max u(c_t^y, c_t^o)$$

2. Subject to the constraint above derived (3)

$$f(k_t) + k_t = (1+n)k_{t+1} + c_t^y + \frac{c_t^o}{1+n}$$

3. The central planner chooses

$$c_t^y, c_t^o, k_{t+1}$$

   that maximizes the welfare of each generation alive in each period of time

4. Notice that the planner does not maximize: c_t^y, c_{t+1}^o, maximizes c_t^y, c_t^o., with $t = 0, 1, 2, ...$

The optimal problem for the social planner (cont.)

1. Formally, the problem is

$$\max_{c_t^y, c_t^o, k_{t+1}} u(c_t^y, c_t^o)$$
$$\textit{subject to}$$
$$f(k_t) + k_t \quad = \quad (1+n)k_{t+1} + c_t^y + \frac{c_t^o}{1+n}$$

2. In the text, this problem is solved by substitution[1], but we will continue to use our Lagrangian method
3. In order to simplify things, let's assume that the central planner is only interested in **steady state values**:

$$k_{t+1} = k_t = k \quad \rightarrow \quad f(k) - nk = c_t^y + \frac{c_t^o}{1+n}$$

[1] The substitution method is solved by the first derivative of $\max u\{c_t^y, \left[(1+n)(f(k) - nk - c_t^y)\right]\}$

First Order Conditions (FOCs)

1. The Lagrangian function is written as

$$\mathcal{L} = u(c_t^y, c_t^o) + \lambda \left[c_t^y + \frac{c_t^o}{1+n} - f(k) + n \cdot k \right]$$

2. The first order conditions are given by

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c^y} &= 0 \rightarrow u'_{c^y} = -\lambda \\
\frac{\partial \mathcal{L}}{\partial c^o} &= 0 \rightarrow u'_{c^o} = -\frac{1}{1+n}\lambda \\
\frac{\partial \mathcal{L}}{\partial k} &= 0 \rightarrow f'(k) = n \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \rightarrow c_t^y + \frac{c_t^o}{1+n} - f(k) + n \cdot k = 0
\end{aligned}$$

Optimal values

1. From the two first FOCs we get

$$\frac{u'_{c^y}}{u'_{c^o}} = 1 + n \tag{4}$$

2. And from the third FOC

$$f'(k) = n \tag{5}$$

3. Conclusion:
   1. the planner finds the capital-labor ratio which is optimal: eq. (5),
   2. then allocates consumption between the young and the old according to: eq (4)

Optimal values (graphically)

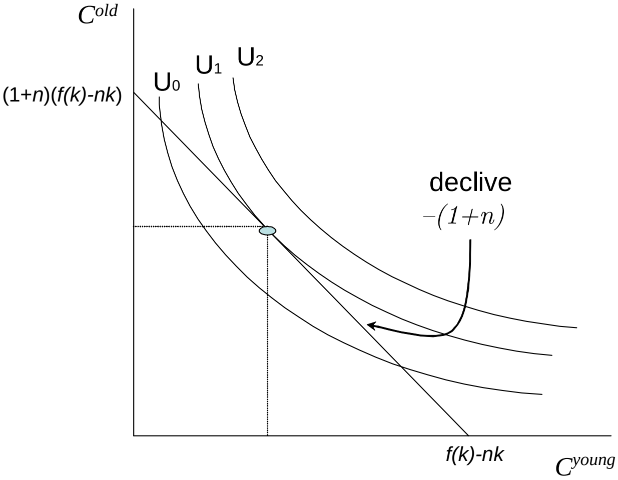


Figure: The social planner's distribution of consumption across generations.

V – Decentralized or competitive allocations

No social planner

1. What happens to the previous results if **there is no social planner?**:
2. Assume that each generation of consumers, and firms, make their choices based on the maximization of their own private interests
   1. Each generation maximize their own utility over time, independently of the welfare of the other generation currently alive
   2. Firms maximize profits
3. Assume that there are no intergenerational transfers
4. What are the optimal values for

$$c_t^y, c_{t+1}^o, k_{t+1}$$

   notice that we have c_t^y, c_{t+1}^o, not c_t^y, c_t^o.
5. We will **NOT** obtain similar values to c^y, c^o, as in the previous section.

Consumers: constraints

1. For each generation born at t, the intertemporal problem is

$$\max_{c_t^y, c_{t+1}^o, s_t} u(c_t^y, c_{t+1}^o)$$

$$\textit{subject to}$$

$$c_t^y + s_t = w_t \tag{6}$$

$$c_{t+1}^o = (1 + r_{t+1}) s_t \tag{7}$$

2. c_t = consumption, s_t = savings, w_t = wage income, r_{t+1} = interest rate.
3. The intertemporal budget constraint can be obtained by cancelling out s_t in the two constraints above

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t$$

4. For each consumer in generation born at t, assume that w_t, r_{t+1} are given.

Rational expectations or perfect foresight

1. On page 27 of Williamson (2006), you find terms like "**rational expectations**" and "**perfect foresight**" (see sentence below).
2. Do not worry about these two terms, they are not extremely important for our argument here.

"The consumer chooses savings and consumption when young and old treating prices, w_t and r_{t+1}, as being fixed. At time t the consumer is assumed to know r_{t+1}. Equivalently, we can think of this as a **rational expectations** *or* **perfect foresight** *equilibrium, where each consumer forecasts future prices, and optimizes based on those forecasts. In equilibrium, forecasts are correct, i.e. no one makes systematic forecasting errors. Since there is no uncertainty here, forecasts cannot be incorrect in equilibrium if agents have rational expectations."*

The Lagrangian function and FOCs

1. The Lagrangian function is

$$\mathcal{L} = u(c_t^y, c_{t+1}^o) + \lambda \left[w_t - c_t^y - \frac{c_{t+1}^o}{1+r_{t+1}} \right]$$

2. And the First Order Conditions (FOCs) are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t^y} &= 0 \rightarrow u'_{c^y} = \lambda \\
\frac{\partial \mathcal{L}}{\partial c_{t+1}^o} &= 0 \rightarrow u'_{c^o} = \frac{1}{1+r_{t+1}} \lambda \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \rightarrow w_t - c_t^y - \frac{c_{t+1}^o}{1+r_{t+1}} = 0
\end{aligned}$$

Consumers's optimal decisions

1. From the two first FOCs we obtain

$$\frac{u'_{c_t^y}}{u'_{c_{t+1}^o}} = 1 + r_{t+1} \tag{8}$$

2. As done before, by using this equation and the third FOC, we can determine the optimal level of

$$\left(c_t^y\right)^*, \left(c_{t+1}^o\right)^*, \left(s_t\right)^*$$

3. For example, we will get something like this for optimal savings

$$s_t = \mathfrak{s}(w_t, r_{t+1}) \tag{9}$$

Williamson's substitution method

Do not worry about this slide

1. We obtained this result

$$\frac{u'_{c_t^y}}{u'_{c_{t+1}^o}} = 1 + r_{t+1}$$

2. Notice that this is totally similar to result obtained by the substitution technique used by Williamson:

$$-u_1\left[w_t - s_t, s_t(1+r_{t+1})\right] + u_2\left[w_t - s_t, s_t(1+r_{t+1})\right](1+r_{t+1}) = 0$$

3. Therefore

$$\begin{aligned} -u_1\left[.,.\right] + u_2\left[.,.\right](1+r_{t+1}) &= 0 \\ \frac{u_1\left[.,.\right]}{u_2\left[.,.\right]} &= 1 + r_{t+1} \end{aligned}$$

4. There we have it: two ways of getting to the same result

$$\frac{u_1\left[.,.\right]}{u_2\left[.,.\right]} = 1 + r_{t+1} \quad , \quad \text{or} \quad \frac{u'_{c^y}}{u'_{c^o}} = 1 + r_{t+1}.$$

The firms' problem

1. The representative firm solves a problem of static optimization
2. The firm produces output with labor (L_t) and capital (K_t) with a production function
$$Q_t = F(K_t, L_t)$$
3. $F(K_t, L_t)$ is linearly homogeneous: doubling inputs, doubles output
4. The representative firm is a price taker in all markets:
$$(w_t, r_t) \text{ as constant in the optimization process}$$
5. Firm's objective: maximize profits

Firms' profits maximization

1. Profits are given by

$$\max_{K_t,L_t}[F(K_t,L_t) - w_t L_t - r_t K_t].$$

2. First order conditions are

$$\begin{aligned} F'_K(K_t,L_t) - r_t &= 0 \\ F'_L(K_t,L_t) - w_t &= 0 \end{aligned}$$

3. As $F(K_t,L_t)$ is linearly homogeneous the two conditions can be written as (see **Note** for more details)

$$f'(k_t) - r_t = 0 \tag{10}$$

$$f(k_t) - k_t \cdot f'(k_t) - w_t = 0 \tag{11}$$

Competitive equilibrium: consumption and savings

Definition

A competitive equilibrium is a sequence of quantities, $\{k_{t+1}, s_t\}_{t=0}^{\infty}$ and a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, which satisfy (i) consumer optimization; (ii) firm optimization; (iii) market clearing; in each period $t = 0, 1, 2, \ldots$, given the initial capital-labor ratio k_0.

1. We have three markets: goods, capital and labor
2. Consumption per capita is given by eq. (8)

$$\frac{u'_{c_t^y}}{u'_{c_{t+1}^o}} = 1 + r_{t+1}$$

3. Savings per capita is given by eq. (9)

$$s_t = \mathfrak{s}(w_t, r_{t+1})$$

4. And what about capital per capita? Let's see.

Competitive equilibrium: capital

1. The supply of capital at $t+1$ is given by the aggregate sum of savings

$$K_{t+1} = s_t \cdot L_t \tag{12}$$

2. The level of capital **per capita** is given by applying a trick to the previous equation:

$$\frac{K_{t+1}}{L_{t+1}} \frac{L_{t+1}}{L_t} = s_t$$

3. Considering that $K_{t+1}/L_{t+1} = k_{t+1}$, and that from eq. (1), $L_{t+1}/L_t = n$, then

$$k_{t+1}(1+n) = s_t$$

Competitive equilibrium: capital (cont.)

1. In the previous slide we obtained

$$k_{t+1}(1+n) = s_t \tag{13}$$

2. From equations (10) and (11), we know that

$$f'(k_t) = r_t \tag{14}$$
$$f(k_t) - k_t \cdot f'(k_t) = w_t \tag{15}$$

3. By substituting for r_t and w_t into eq. (13) we obtain a difference equation for $\{k_t\}_{t=1}^{\infty}$

$$k_{t+1} = \mathcal{F}(k_t)$$

   the solution of which depends upon $\mathcal{F}$ and k_0.

4. Notice that now we have the competitive equilibrium determined:

$$\{k_{t+1}, s_t, w_t, r_t, c_t^y, c_{t+1}^o\}_{t=1}^{\infty}$$

VI – Example

Example

1. Assume an economy with the following equations describing the behavior of each generation of consumers and firms

$$\begin{aligned} u(c_t^y, c_{t+1}^o) &= \ln c_t^y + \beta \ln c_{t+1}^o \\ c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} &= w_t \\ F(K_t, L_t) &= \gamma K_t^\alpha L_t^{1-\alpha} \end{aligned}$$

   with $\beta > 0$, $\gamma > 0$, and $0 < \alpha < 1$

2. Calculate the values of $\{k_{t+1}, s_t\}_{t=0}^{\infty}$ *and* $\{w_t, r_t\}_{t=0}^{\infty}$ compatible with the competitive equilibrium.
3. Compare those values with those that satisfy the Pareto optimum.

Solution: competitive equilibrium consumption

1. A typical generation maximizes intertemporal consumption.
2. The Lagrangian comes

$$\mathcal{L} = \ln c_t^y + \beta \ln c_{t+1}^o + \lambda (w_t - c_t^y - \frac{c_{t+1}^o}{1 + r_{t+1}})$$

3. From the Euler equation we obtain

$$u'_{c_t^y} = \beta(1 + r_{t+1}) u'_{c_{t+1}^o}$$

4. Therefore

$$c_{t+1}^o = \beta(1 + r_{t+1}) c_t^y$$

5. Using the previous eq. together with the intertemporal budget constraint we obtain

$$\begin{aligned} c_{t+1}^o &= \frac{\beta(1 + r_{t+1})}{1 + \beta} w_t \\ c_t^y &= \frac{1}{1 + \beta} w_t \end{aligned}$$

Solution: competitive equilibrium savings

1. Let's solve for each generation's savings.
2. From the first period budget constraint (eq. 6) we know that

$$
\begin{aligned}
s_t &= w_t - c_t^y \\
&= w_t - \frac{1}{1+\beta} w_t
\end{aligned}
$$

3. That is

$$s_t = \frac{\beta}{1+\beta} w_t \tag{16}$$

Solution: competitive equilibrium capital

1. The production function is

$$F(K_t, L_t) = \gamma K_t^{\alpha} L_t^{1-\alpha}$$

2. The marginal product of capital is given by differentiating $F(K_t, L_t)$ with respect to K

$$r_t = F'_K = \alpha\gamma K_t^{\alpha-1} L_t^{1-\alpha} = \alpha\gamma \frac{K_t^{\alpha-1}}{L_t^{\alpha-1}} = \alpha\gamma k_t^{\alpha-1}$$

3. The marginal product of labor is given by differentiating $F(K_t, L_t)$ with respect to L

$$w_t = F'_L = (1-\alpha)\gamma K_t^{\alpha} L_t^{1-\alpha-1} = (1-\alpha)\gamma \frac{K_t^{\alpha}}{L_t^{\alpha}} = (1-\alpha)\gamma k_t^{\alpha}$$

4. Therefore, we have got

$$r_t = \gamma\alpha k_t^{\alpha-1} \quad (17)$$

$$w_t = (1-\alpha)\gamma k_t^{\alpha} \quad (18)$$

Solution: competitive equilibrium capital (cont.)

1. From equation (13) we know that

$$k_{t+1}(1+n) = s_t$$

2. And from eq. (16) we know that

$$s_t = \frac{\beta}{1+\beta} w_t$$

3. And from eq. (18) we know that

$$w_t = (1-\alpha)\gamma k_t^{\alpha}$$

4. Therefore

$$k_{t+1}(1+n) = \frac{\beta}{1+\beta} \underbrace{(1-\alpha)\gamma k_t^{\alpha}}_{w_t}$$

Solution: competitive equilibrium capital (cont.)

1. We obtained a nonlinear difference equation for the solution of the capital stock per capita

$$k_{t+1}(1+n) = \frac{\beta}{1+\beta}(1-\alpha)\gamma k_t^{\alpha}$$

2. Let's rewrite this equation in a simpler way

$$k_{t+1} = \psi k_t^{\alpha}$$

   where $\psi = \frac{(1-\alpha)\gamma\beta}{(1+n)(1+\beta)}$.

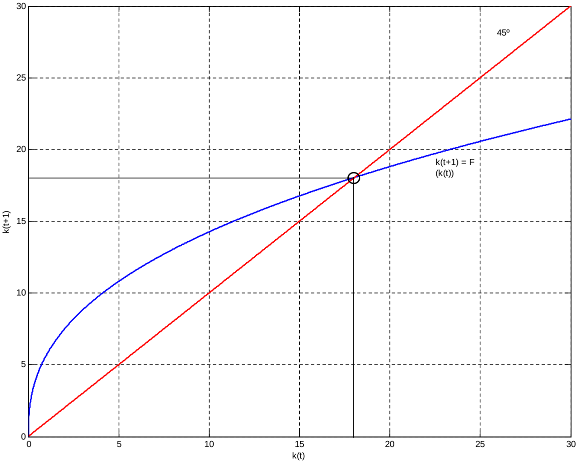
3. The solution for the steady state is obtained by imposing the usual condition $k_{t+1} = k_t = k^*$

$$k^* = (\psi)^{\frac{1}{1-\alpha}}$$

4. See next figure.

Solution: competitive equilibrium capital (cont.)

1. Assume: $\alpha = 0.4$, $\gamma = 20$, $\beta = 0.95$, $n = 0.03$
2. The competitive capital stock per capita is: $k^* = 18.06$



Solution: socially optimal equilibrium

1. As seen in eq (5) — section IV — the socially optimal value of capital per capita is given by
$$f'(k) = n$$
2. Therefore, as $f(k) = \gamma k^{\alpha}$, see eq. (**??**), then
$$f'(k) = \gamma \alpha k_t^{\alpha - 1}$$
3. Using the two previous equations and calling the socially optimal level of capital per capita as $\hat{k}$, we have
$$\gamma \alpha \hat{k}_t^{\alpha - 1} = n$$
4. Or
$$\hat{k} = \left(\frac{\gamma \alpha}{n} \right)^{\frac{1}{1-\alpha}} \qquad (19)$$

Comparing the two solutions

1. Our parameter values are:

$$\alpha = 0.4, \gamma = 20, \beta = 0.95, n = 0.03$$

2. The **socially optimal** level of capital per capita is

$$\hat{k} = \left(\frac{\gamma\alpha}{n}\right)^{\frac{1}{1-\alpha}} = \left(\frac{20 \times 0.4}{0.03}\right)^{\frac{1}{1-0.4}} = 11048$$

3. We saw that the **competitive level** of capital per capita was

$$k^* = 18.06$$

4. Therefore

$$\hat{k} > k^*$$

5. Conclusion: the competitive equilibrium is not socially optimal. Without public intervention, the economy ends up with too little capital. **DYNAMIC INEFFICIENCY**.

Making the two solutions equal

1. If the government wants to correct this inefficiency, what is at its disposal?

Taxes on the young generation

Transfers to the old generation

2. The problem is to find the debt policy, determined by b, which yields a competitive equilibrium steady state tha is socially optimal
3. We want to find $\hat{b}$ such that $k^*(\hat{b}) = \hat{k}$.
4. This is discussed in section 2.6 of Williamson (2006), but we will not discuss further this particular problem here.
5. Instead we will concentrate on which social security system looks better in the world we currently live.

VII – Sustainability of Social Security Systems

The two social security systems

1. Currently the main social security systems that are available in the world are:
    1. Fully funded system
    2. Pay-As-You-Go (PAYG)
2. **Fully–Funded**. The young generation at t, consume and saves at t, invests financially its savings in $t+1$, in order to finance its consumption at $t+1$, as it receives no wage income in this period.
3. **PAYG**. The fundamental difference is that the savings of the young generation at t are used to finance the consumption of the old generation at t.

The two social security systems (cont)

1. What are the crucial factors that affect the sustainability of such systems?
2. A PAYG depends crucially upon demographic problems:
    1. the **number of people** that save, relatively to the number of people that are retired
    2. the **extension of the period** where savings occur, relatively to the period of retirement
3. A fully–funded system does not depend upon demographic problems, but depends crucially upon:
    1. The **rate of return** to financial investments
    2. The **risk** associated with very, very long term financial investments

An economy without social security

1. The representative consumers of generation born at t maximizes

$$\max_{c_t^y, c_{t+1}^o, s_t} u(c_t^y, c_{t+1}^o)$$

2. Subject to the usual constraints (y_t is income, wage income)

$$\begin{aligned} c_t^y + s_t &= y_t \\ c_{t+1}^o &= (1 + r_{t+1}) s_t \end{aligned}$$

3. Assume that the population (N) grows at a constant annual rate n

$$N_{t+1} = (1 + n) N_t$$

4. Now assume that utility is logarithmic

$$u(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$$

5. For simplicity assume: $\beta = 1$.

An economy without social security (cont.)

1. The Lagrangian comes out as

$$L = \ln c_t^y + \ln c_{t+1}^o + \lambda \left(y_t - c_t^y - \frac{c_{t+1}^o}{1 + r_{t+1}} \right)$$

2. From the Euler equation and the consolidated intertemporal constraint we obtain

$$\begin{aligned} c_t^y &= \frac{1}{2} y_t \\ c_{t+1}^o &= \frac{(1 + r_{t+1})}{2} y_t \end{aligned}$$

3. Now using the first period budget constraint and $c_t^y = \frac{1}{2} y_t$, the optimal savings is given by

$$s_t = \frac{1}{2} y_t$$

A Fully–Funded System

1. Under a fully-funded pension system, contributions are invested in the financial markets and receive an annual return given by

$$r_{t+1}$$

2. The government imposes a proportional payroll tax to finance pensions

$$contributions = \tau \cdot y_t$$

3. The financial return to this generation when old $(t+1)$ is given by

$$return = (1 + r_{t+1})\,(\tau \cdot y_t)$$

4. So the two budget constraints are given by

$$\begin{aligned} c_t^y + s_t &= (1-\tau)\,y_t \\ c_{t+1}^o &= (1+r_{t+1})s_t + (1+r_{t+1})\,(\tau \cdot y_t) \end{aligned}$$

A Fully–Funded System (cont.)

1. The Lagrangian is given by

$$L = \ln c_t^y + \ln c_{t+1}^o + \lambda \left[(1-\tau)\, y_t + \tau y_t - c_t^y - \frac{c_{t+1}^o}{1+r_{t+1}} \right]$$

2. From the Euler equation and the consolidate intertemporal budget constraint we get

$$c_t^y = \frac{1}{2} y_t \qquad (20)$$

$$c_{t+1}^o = \frac{1+r_{t+1}}{2} y_t \qquad (21)$$

3. And the optimal savings are

$$s_t = \frac{1}{2} y_t - \tau \cdot y_t \qquad (22)$$

4. **Funny thing:** the optimal values for a fully-funded system are exactly the same as those of a no-social-security system.

A Pay-As-You-Go System

1. The government imposes a proportional payroll tax to finance pensions
$$contributions = \tau \cdot y_t$$
2. The financial return to this generation when old $(t+1)$ does not depend in this case on r_{t+1}, as the contributions are not invested in the financial markets
3. Now the return to the old generation will depend on the balance sheet of social security system.
4. In every period t, the returns to the **old** generation $(b_t \cdot N_t)$ have to be equal to the contributions of the **young** generation $(N_{t+1} \cdot \tau y_{t+1})$

$$N_{t+1} \cdot \tau y_{t+1} = b_t \cdot N_t \tag{23}$$

where b_t is the retirement benefit for a person of the old generation alive at t.

A Pay-As-You-Go System (cont.)

1. Now consider two important assumptions: population and income per capita grow at constant rates (respectively, n and g)

$$\begin{aligned} N_{t+1} &= (1+n)\,N_t \\ y_{t+1} &= (1+g)y_t \end{aligned}$$

2. So eq.(23) — the balance of the PAYG system — can be rewritten as

$$b_t = (1+n)\,(1+g)\,\tau y_t$$

3. Therefore, the return per person of the old generation is given by

$$(1+n)\,(1+g)$$

while

$$\tau y_t$$

is the contribution per person of the young generation.

A Pay-As-You-Go System (cont.)

1. Now, for each generation, the two constraints are as follow

$$\begin{aligned} c_t^y + s_t &= (1-\tau)\, y_t \\ c_{t+1}^o &= (1+r_{t+1}) s_t + (1+n)\,(1+g)\,\tau y_t \end{aligned}$$

2. The Lagrangian will come as

$$\begin{aligned} L &= \ln c_t^y + \ln c_{t+1}^o + \\ & \lambda \left[(1-\tau)\, y_t + \frac{(1+n)\,(1+g)\,\tau y_t}{1+r_{t+1}} - c_t^y - \frac{c_{t+1}^o}{1+r_{t+1}} \right] \end{aligned}$$

A Pay-As-You-Go System (cont.)

1. From the Euler equation and the consolidated intertemporal budget constraint we obtain

$$c_t^y = \frac{1}{2}y_t + \frac{(1+n)(1+g)-(1+r_{t+1})}{2(1+r_{t+1})}\tau y_t \quad (24)$$

$$(25)$$

$$c_{t+1}^o = \frac{1+r_{t+1}}{2}y_t + \frac{(1+n)(1+g)-(1+r_{t+1})}{2}\tau y_t$$

2. And the savings function is given by

$$s_t = \frac{1}{2}y_t - \tau y_t - \frac{(1+n)(1+g)-(1+r_{t+1})}{2(1+r_{t+1})}\tau y_t \quad (26)$$

3. Notice: what happens in the three results above if

$$(1+n)(1+g) = (1+r_{t+1})$$

Comparing a PAYG with a Fully Funded System

1. If: $(1+n)(1+g) = (1+r_{t+1})$ the two systems provide exactly the same welfare

$$\begin{aligned} \left(c_t^y, c_{t+1}^o\right)^*_{PAYG} &= \left(c_t^y, c_{t+1}^o\right)^*_{Funded} \\ (s_t)^*_{PAYG} &= (s_t)^*_{Funded} \end{aligned}$$

2. If $(1+n)(1+g) > (1+r_{t+1})$ the PAYG is better

$$\begin{aligned} \left(c_t^y, c_{t+1}^o\right)^*_{PAYG} &> \left(c_t^y, c_{t+1}^o\right)^*_{Funded} \\ (s_t)^*_{PAYG} &> (s_t)^*_{Funded} \end{aligned}$$

3. In this case, **all generations** (now alive or those to be born in the future) will benefit from the implementation of a PAYG system.

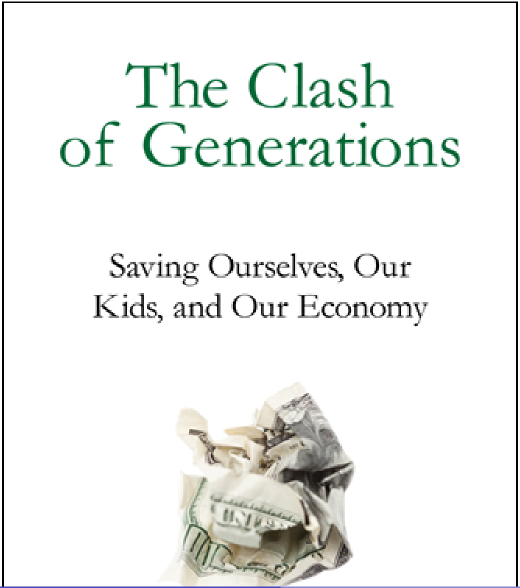
Comparing a PAYG with a Fully Funded System

1. If: $(1+n)(1+g) < (1+r_{t+1})$ the fully funded system is better

$$\begin{aligned} \left(c_t^y, c_{t+1}^o\right)_{PAYG}^* &< \left(c_t^y, c_{t+1}^o\right)_{Funded}^* \\ (s_t)_{PAYG}^* &< (s_t)_{Funded}^* \end{aligned}$$

2. The terrible problem: this creates a serious intergenerational problem.
3. If there is a perception that n declines over time (or what is similar, the old generation experiences retirement periods that increase over time), or both, while g does not increase over time, the young generation **knows** that the system will blow up at a certain moment.
4. The young generation will not accept to be worse off then their parents, and will certainly not accept also that their children are doomed to live worsen than they have lived.
5. We have a very serious economic, financial and political problem to solve. Remember the Kotlikoff & Burns message: **The Clash of Generations**.

Laurence J. Kotlikoff and Scott Burns



The United States is bankrupt, flat broke. Thanks to accounting that would make Enron blush, America's insolvency goes far beyond what our leaders are disclosing. The United States is a fiscal basket case, in worse shape than the notoriously bailed-out countries of Greece, Ireland, and others. How did this happen? In The Clash of Generations, experts Laurence Kotlikoff and Scott Burns document our six-decade, off-balance-sheet, unsustainable financing scheme. They explain how we have balanced our longer lives on the backs of our (relatively few) children. At the same time, we've been on a consumption

Final comments

1. Notice that the results in our analysis may be changed if some hypotheses are introduced.
2. We worked out how an individual consumer will adjust savings in the two different systems, **taking incomes and interest rates as given**.
3. In the real world, we have to allow for the possibility that **the social security system might affect aggregate savings and investment**, which in turn would have an effect on incomes and interest rates.
4. Example 1. **Procrastination: too low savings**. A fully-funded system is similar to a no-social-security system. So if people are left to themselves to make decisions about their future well-being, they may decide to lower their savings ... and we are left with another type of problem.
5. Example 2. **Too much risk**. What happens if people save and invest their savings in highly risky investments. Bankruptcy brings another type of problem.
6. Other examples. **Hyperinflation, wars**.

VIII - Bibliography

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