The Inflation Game: Rules vs Discretion (Current Controversies in Monetary Policy)

— Questions and SOLUTIONS —

Modern Macroeconomics — ISCTE–IUL — May 2014

Problem A. Assume that the Central Bank's loss function is given by the function:

$$L = \phi\,(u - u^*) + \gamma\,(\pi - \pi^*)^2$$

u is the unemployment rate, π is the inflation rate, and ϕ and γ are parameters. The asterisk represents the desired values for each variable.
We know that $\phi = 0.25$, $\gamma = 4$ and and that the behavior of the supply side of the economy can be described by the following Phillips curve:

$$u = u^n - \alpha(\pi - \pi^e)$$

where u^n is the natural level of unemployment, π^e is the level of expected inflation, and $\alpha = 1.5$. Finally assume that private agents have rational expectations

$$\pi^e = \pi.$$

1. Explain the choices of the central bank that are represented in its loss function above.
2. Assume that the central bank desired level for inflation is 0%. Determine the level of optimal inflation in the case of discretionary behavior by the central bank.
3. Determine the same as in the previous question, but now having the central bank displaying commitment to maintain inflation at the level of its natural rate.
4. Explain why the result in (3) is better than the result in (2).
5. Assume that due to some external force the central bank is obliged to act under discretion (no commitment). What is the option available to the central bank in order to achieve zero inflation? Explain.

Solutions A.

1. There are three major points that should be highlighted by the loss function of the central bank:

1. The central bank is concerned about both inflation rate deviations from target $(\pi - \pi^*)$ and unemployment rate deviations from target $(u - u^*)$.

2. The central bank gives too little importance to the unemployment deviations ($\phi = 0.25$) relatively to the importance it attaches to achieving the target of inflation ($\gamma = 4$)

3. Notice that in the case of inflation, the central bank is worried both about positive and negative deviations from this target. This is given by the fact that $(\pi - \pi^*)^2$ gives the same loss for $\pi - \pi^* = +2$ or for the case where $\pi - \pi^* = -2$. However, in the case of unemployment deviations, the central bank only experiences losses when $u - u^* > 0$. If $u - u^* < 0$ the central bank is "very happy", it does not "lose" anything.

2. Define the gap between desired and natural unemployment rates as

$$k \equiv u^* - u^n.$$

Now pick up the Phillips curve $u = u^n - \alpha(\pi - \pi^e)$. Subtract u^* from both sides

$$u - u^* = u^n - u^* - \alpha(\pi - \pi^e)$$

Now the term $u^n - u^*$, by the way we defined k above, is equal to $u^n - u^* = -k$. Therefore the Phillips curve can be written as

$$u - u^* = -k - \alpha(\pi - \pi^e) \tag{1}$$

Substitute eq. (1) into the Loss function of the central bank and notice that $\pi^* = 0$. Then we get[1]

$$L = \phi\underbrace{[-k - \alpha(\pi - \pi^e)]}_{=u-u^*} + \gamma\,(\pi)^2 \tag{2}$$

We know that with discretion π^e is treated as constant by the central bank in its optimization process, and so (using d subscripts for discretion) we get

$$\frac{\partial L}{\partial \pi_d} = 0 \Longrightarrow -\phi\alpha + 2\gamma\pi_d = 0$$

From which it is easy to obtain the optimal level for inflation under discretion as

$$\pi_d = \frac{\phi\alpha}{2\gamma} = \frac{0.25 \times 1.5}{2 \times 4} = 0.046.$$

[1] Do not forget that $\pi^* = 0$.

3. Notice that with commitment, we know that the expectations of the private agents are endogenously treated by the central bank. And as the bank knows that these agents have rational expectations

$$\pi^e = \pi$$

just by inserting this condition into the Loss function (eq. 2) we get

$$\begin{aligned} L &= \phi[-k - \alpha\underbrace{(\pi - \pi)}_{=0}] + \gamma\,(\pi)^2 \\ &= \phi[-k] + \gamma\,(\pi)^2 \end{aligned}$$

Using c subscript for commitment, we get

$$\frac{\partial L}{\partial \pi_c} = 0 \Longrightarrow 2\gamma\pi_c = 0$$

from which

$$\pi_c = 0$$

4. The result in question 3 is better than the result in 2 because the central bank is committed to achieve its target of zero inflation ($\pi^* = 0$), and private agents know this and have rational expectations. If the central bank commitment is credible to private agents, they should formulate expectations about inflation such that $\pi^e = \pi_c = 0$.

5. Notice that under discretion above, the optimal level of inflation was given by

$$\pi_d = \frac{\phi\alpha}{2\gamma}.$$

What happens if the central bank, under discretion, is not concerned at all with unemployment deviations from target ($\phi = 0$)? The answer is:

$$\pi_d = 0.$$

That's the justification that some people are using to defend that a central bank should, under any circumstances, be worried only about inflation; not about anything else.

Problem B. Assume that the Central Bank's loss function is given by the quadratic function:

$$L = u^2 + \gamma\pi^2$$

u is the unemployment rate, γ is a parameter, and π is the inflation rate. We know that $\gamma = 2.5$ and that the behavior of the supply side of the economy can be described by the following Phillips curve:

$$u = u^n - \alpha(\pi - \pi^e)$$

where u^n is the natural level of unemployment, π^e is the level of expected inflation, and $\alpha = 15$. Finally assume that private agents have rational expectations

$$\pi^e = \pi.$$

1. Determine the level of optimal inflation in the case of discretionary behavior by the central bank.

2. Determine the same as in the previous question, but now having the central bank displaying commitment to maintain inflation at the level of its natural rate.

3. Explain why the result in (2) is better than the result in (1).

4. Explain either by your own words, or by some sophisticated approach, what would happen in both scenarios above, if private agents had adaptive expectations instead of rational expectations. (not covered this year, BUT FOR THOSE WHO KNOW THE ESSENCE OF ADAPTIVE EXPECTATIONS, THE ANSWER IS EASY. With adaptive expectations, private agents formulate expectations using only information from the past, so they do not take into consideration any target for inflation that the central bank may announce)

Solutions B.

1. This problem is totally similar to the previous one. The only difference resides in the particular form of the Loss function.

Performing similar steps,[2] the Loss function can be written as

$$L = [u^n - \alpha(\pi - \pi^e)]^2 + \gamma (\pi)^2$$

Therefore

$$\frac{\partial L}{\partial \pi_d} = 0 \Longrightarrow 2\,[u^n - \alpha(\pi_d - \pi^e)] \times (-\alpha) + 2\gamma\pi_d = 0$$

This condition can be written as

$$-\alpha\,[u^n - \alpha(\pi_d - \pi^e)] + \gamma\pi_d = 0.$$

and just by getting rid off the parentheses above gives

$$-\alpha u^n + \alpha^2\pi_d - \alpha^2\pi^e + \gamma\pi_d = 0.$$

[2] This case is actually simpler because the k definition is not necessary here, given the particular form of the Loss function.

Now impose the condition that private agents have rational expectations

$$\pi^e = \pi$$

and solve for π_d . The result is surprisingly simple

$$\pi_d = \frac{\alpha \times u^n}{\gamma} = \frac{15}{2.5} u^n.$$

2. The solution for commitment is immediate. The Loss function comes out as

$$L = [u^n]^2 + \gamma (\pi)^2$$

and therefore

$$\frac{\partial L}{\partial \pi_c} = 0 \Longrightarrow 2\gamma \pi_c = 0$$

from which

$$\pi_c = 0$$

3. See answer 4 in the previous problem.

4. NOT TO BE ANSWERED. NOT COVERED IN CLASSES.

For the curious student, here is a sketch of the answer. If private agents have adaptive expectations (in which they take into account information from the past in their decision making), it is irrelevant for them whether the central behaves accordingly to discretion or commitment. Imagine that the central bank announces today a new target for the inflation rate. If they have adaptive expectations, private agents will not take into account this new information. Therefore, the final result will be just equal under commitment and under discretion. What sense does it make for the central bank to commit to a certain target, if private agents ignore this new information?

Problem C. Assume that the Central Bank's loss function is given by the following function:

$$L = \beta (u - u^*)^2 + \gamma (\pi - \pi^*)^2$$

u is the unemployment rate, γ, β are parameters, and π is the inflation rate. An asterisk is used to represent the central bank's desired values for each variable.
We know that the behavior of the supply side of the economy can be described by the following Phillips curve:

$$u = u^n - \alpha(\pi - \pi^e)$$

where u^n is the natural level of unemployment, π^e is the level of expected inflation, and $\alpha = 15$. Finally assume that private agents have rational expectations

$$\pi^e = \pi.$$

1. Explain the logic behind the Loss function above, as far as the targets of the central bank are concerned.

2. Assuming that $u^* = 4, \pi^* = 0$, determine the level of optimal inflation in the case of discretionary behavior by the central bank.

3. Determine the same as in the previous question, but now having the central bank displaying commitment to maintain inflation at the level of its natural rate.

4. Explain why the result in (2) is better than the result in (1).

5. What is the condition that should hold in order to have the same result in both scenarios: discretion and commitment. Explain.

Solutions C.

This problem is totally similar to the presentation that is contained in the slides for this material.

Notice that regarding Problem A, the only difference resides in the Loss function. In the present case, both deviations are raised to the power of 2:

$$L = \beta (u - u^*)^2 + \gamma (\pi - \pi^*)^2$$

and not just the term $(\pi - \pi^*)$ as in Problem A.

Very simple, try it yourself. Actually, it is just a copy of the slides.