Overlapping–Generations models

Problems

Modern Macroeconomics — ISCTE–IUL — May 2014

Problem A. Using the Overlapping-Generations model, where c_t^y, c_{t+1}^o represent the level of consumption of one given generation, respectively, when it is young and when it is old, and c_1^o is the level of consumption of the first (initial) old generation, answer the following questions:

1. Define one allocation that satisfies the Pareto optimum.
2. Assume that the total revenue in each period is given by $TR = 20$. Assume also that the utility function of each generation is given by

   $$u(c_t^y, c_{t+1}^o) = (3/4)\, c_t^y + c_{t+1}^o$$

   From the three following allocations, which one corresponds to the best possible allocation of resources over time?

   (a) Allocation A: $(c_t^y = 10, c_{t+1}^o = 10)$
   (b) Allocation B: $(c_t^y = 12, c_{t+1}^o = 8)$
   (c) Allocation C: $(c_t^y = 8, c_{t+1}^o = 12)$

3. Which tool can be used such that all allocations would satisfy the Pareto Optimum. Explain.

Problem B. Consider the following overlapping generations growth model. Time is indexed by $t = 0, 1, 2, ...$, and in period t there are L_t two-period-lived consumers born, where $L_1 = (1+n)^1 L_0$, with L_0 given and $n > 0$. In periods $t = 0, 1, 2, ...$, each young consumer is endowed with y units of the consumption good. Each old consumer (including the initial old in period 0) is endowed with nothing.

There is a given technology which permits one unit of period t consumption goods to be converted to $1 + r$ units of period $t + 1$ consumption goods, for $t = 0, 1, 2, ...$ In period t, the government collects a lump-sum tax of τ_t^y from each young consumer, and a lump-sum tax of τ_t^o from each old consumer.

A consumer born in period t has preferences given by

$$u(c_t^y, c_{t+1}^o) = 2(c_t^y)^{1/2} + 2(c_{t+1}^o)^{1/2}$$

1. Write down the government's budget constraint. (Hint: the only source of government revenue is the lump-sum taxes imposed upon each generation)

2. Suppose a *pay-as-you-go* social security scheme, where each period the government taxes the young so as to make transfers to the old. That is, the government sets $\tau_t^y = \tau$ for $t = 0, 1, 2, ...$ Determine the effects of an increase in τ on the savings of each young consumer, and on the welfare of each generation.

3. Determine also the Pareto optimal level of τ and explain your results. (Hint: this should depend whether $n > r$, or vice versa)

4. Suppose a *fully-funded* social security system where the government taxes the young in period t, puts the proceeds of the tax into storage, and then makes transfers to the old in period $t + 1$ with the proceeds from period t storage. In this case the government sets $\tau_t^y = \tau$ for $t = 0, 1, 2, ...$ Determine the effects of an increase in τ on the savings of each young consumer, the consumption of the young and the old in each generation, and on the welfare of each generation.

5. As in question (3), determine the Pareto optimal level of τ and explain your results.