# Ricardian Equivalence
# Exercises

with solutions

**Modern Macroeconomics — ISCTE–IUL, April 2014**

## Exercise 1

Use the two-period model of the representative household discussed in the slides. Consider now that the government, in order to finance its expenditures, imposes a proportional tax on the level of income in each period (wich is exogenously determined). This proportional tax is given by $\tau$ in each period. The intertemporal decision making problem for our representative agent can be written down as

$$\begin{aligned} U &= u(c_0) + \beta u(c_1) \\ c_1 + a_1 + \tau_1 y_1 &= y_1 + (1 + r_0)a_0 \\ c_2 + a_2 + \tau_2 y_2 &= y_2 + (1 + r_1)a_1 \end{aligned}$$

where $U$ is life-time utility, $\beta$ is the subjective discounting rate of future utility, $c_t$ is consumption in period $(t = 1, 2)$ of the agent's life, $y_t$ is the (exogenous) income in period $t$, and $a_t$ is financial assets possessed by the household in period $t$.

Assume that the household saves in the first period of life in order to enjoy a pleasant retirement in the second period of life. Assume furthermore that the utility (or "felicity") function takes the following form

$$u(c_t) = \ln c_t$$

1. Interpret the model and derive the lifetime budget equation. Explain what you assume about $a_2$.
2. Introduce the government and demonstrate Ricardian equivalence.
3. Compute the expressions for optimal consumption and savings plans (i.e. $c_1$, $c_2$, and $s_1 \equiv a_1 - a_0$).
4. Assume that there is a broad income tax (which also taxes interest income). Redo part (3). Show how consumption and saving depend on the income tax rate.

## Exercise 2

Consider an economy that lasts for 2 periods $t = 1, 2$.The economy is populated by a large amount of households, all equal, each one with preferences

$$u(c_1, c_2) = \ln c_1 + \beta \ln c_2$$

where $\beta < 1$ is the discount factor, $(c_1, c_2)$ is consumption in the two periods. Each household is endowed with income $(y_1, y_2)$ and can save/borrow an amount $a_1$ between time 1 and 2 at the interest rate $r$. They face taxes on capital income $\tau_2$ in the second period but they do not pay any tax in the first period. Thus, the households' budget constraints in the two periods are

$$\begin{aligned} c_1 + a_1 &= y_1 \\ c_2 + \tau_2 (r_1 a_1) &= y_2 + (1 + r_1) a_1 \end{aligned}$$

The government has expenditures $(g_1, g_2)$ in the two periods, financed with capital income taxes $\tau_2$ in the second period and debt $b_1$ in the first period. At the end of the two periods, the Government has to pay back its debt, gross of interests, only through taxes.

1. Write the first and second period budget constraint for the government and the intertemporal budget constraint for the government .
2. Solve the problem of the household and derive the first-order conditions for consumption in both periods $(c_1, c_2)$. Use these conditions to derive the Euler equation. Does the Euler Equation depend on taxes $\tau_2$?
3. State the meaning of Ricardian Neutrality in this economy.
4. Does Ricardian Neutrality hold in this economy? Explain your answer, possibly proving your result.

Solutions to

Ricardian Equivalence Exercises

"Modern Macro"

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Exercise 1

1. The lifetime budget constraint for our representative household is give by cancelling out $\underline{a_1}$ in the two period constraints and also by assuming

$$a_2 = 0$$

Otherwise the representative agent would die with positive financial assets and would not maximize his/her utility. By doing so, we get

$$c_1 + \frac{c_2}{1+r_1} = \overbrace{(1+r_0)a_0 + y_1 + \frac{y_2}{1+r_1}}^{\Omega}. \quad (1)$$

2. The two period budget constraints for the Government are given by (assuming that the Government imposes taxes and issues bonds):

$$g_1 + r_0 b_0 = \tau_1 y_1 + \overbrace{\Delta \text{bonds}}^{b_1 - b_0}$$

$$g_2 + r_1 b_1 = \tau_2 y_2 + \underbrace{\Delta \text{bonds}}_{b_2 - b_1}$$

By setting $b_2 = 0$ (in order to rule out a Ponzi game), we get the following intertemporal government constraint by cancelling out $b_1$ above:

$$g_1 + \frac{g_2}{1+r_1} + (1+r_0) b_0 = \tau_1 y_1 + \frac{\tau_2 y_2}{1+r_1} \,. \quad (2)$$

Now, we know that with only households and a Government in our model economy, this must hold

$$b_0 = a_0 \qquad (3)$$

Then, by inserting (3) and (2) into (1) we get

$$c_1 + \frac{c_2}{1+r_1} = y_1 - g_1 + \frac{y_2 - g_2}{1+r_1}. \quad (4)$$

that is, the tax parameters drop out and the path of taxes do not matter for the dynamics of this economy.

<u>Ricardian Equivalence holds</u>.

3. Setting the Lagrangian

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \left[ \Omega - c_1 - \frac{c_2}{1+r_1} \right]$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = 0 \Rightarrow \frac{1}{c_1} = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = 0 \Rightarrow \frac{\beta}{c_2} = \frac{\lambda}{1+r_1}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \Omega - c_1 - \frac{c_2}{1+r_1} = 0$$

Now, use the two first order conditions and get the Euler Equation

$$c_2 = \beta(1+r_1)c_1 . \qquad (5)$$

Now, combining (5) and (1) we obtain

$$c_1^* = \frac{\Omega}{1+\beta}$$

$$c_2^* = \frac{\beta(1+r_1)\Omega}{1+\beta} .$$

In order to determine the level of savings

$$\begin{aligned} s_1 &= a_1 - a_0 \\ &= r_0 a_0 + (1-\tau_1)y_1 - c_1^* \\ &\vdots \\ &= r_0 a_0 + (1-\tau_1)y_1 - \frac{\Omega}{1+\beta} . \end{aligned}$$

4. If interest income is taxed, then the household budget constraint becomes

$$c_1 + \frac{c_2}{1+r^*} = [1 + r(1-\tau_1)]a_0 + (1-\tau_1)y_1 + \frac{(1-\tau_2)y_2}{1+r^*} = \Omega^*$$

where $r^* = r_1(1-\tau_2)$.

Applying the same steps, we will obtain

$$c_1^* = \frac{\Omega^*}{1+\beta}$$

$$c_2^* = \frac{\beta(1+r^*)\,\Omega^*}{1+\beta}.$$

So, results are similar: $c_1^*$ and $c_2^*$ depend on net income. The only difference is that now we have net income also affected by interest income.

# Exercise 2

1. The first and second period budget constraints for the Government are given by:

$$g_1 = b_1 \qquad (1)$$
$$g_2 + (1+r_1)\, b_1 = J_2(a_1 r_1) .$$

By cancelling out $b_1$ in system (1), we obtain the intertemporal budget constraint for the Government:

$$g_1 + \frac{g_2}{1+r_1} = \frac{J_2(a_1 r_1)}{1+r_1} . \qquad (2)$$

2. To maximize life time utility of our representative household, we need first to obtain the intertemporal budget constraint from the two period constraints. We do this by cancelling out $a_1$ in these two constraints.

(see next)

From the $1^{st}$ period constraint we know that

$$a_1 = y_1 - c_1 \,, \qquad (3)$$

and from the second period constraint

$$c_2 + \tau_2 (r_1 a_1) = y_2 + (1+r_1) a_1$$

we may obtain

$$c_2 = y_2 + (1+r_1)a_1 - (\tau_2 r_1) a_1$$

$$c_2 = y_2 + \left[ (1+r_1) - \tau_2 r_1 \right] a_1$$

$$c_2 = y_2 + \left[ 1 + (1-\tau_2) r_1 \right] a_1 \,. \qquad (4)$$

Now, insert (3) into (4) and we will obtain the intertemporal budget constraint for the representative household as

$$c_1 + \frac{c_2}{1+(1-\tau_2)r_1} = y_1 + \frac{y_2}{1+(1-\tau_2)r_1} \,. \qquad (5)$$

In order to proceed, let's write down the Lagangian function, and using eq. (5)

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \underbrace{\left[ y_1 + \frac{y_2}{1+(1-\tau_2)r_1} - c_1 - \frac{c_2}{1+(1-\tau_2)r_1} \right]}_{eq(5)=0}$$

The first order conditions are straightforward:

$$\frac{\partial \mathcal{L}}{\partial c_1} = 0 \Rightarrow \frac{1}{c_1} - \lambda = 0 \Rightarrow \frac{1}{c_1} = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = 0 \Rightarrow \frac{\beta}{c_2} - \frac{\lambda}{1+(1-\tau_2)r_1} = 0$$

$$\Rightarrow \frac{\beta\left[1+(1-\tau_2)r_1\right]}{c_2} = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow y_1 + \frac{y_2}{1+(1-\tau_2)r_1} - c_1 - \frac{c_2}{1+(1-\tau_2)r_1} = 0$$

By using the two first order conditions above, we obtain the Euler Equation:

$$c_2 = \beta\left[1+(1-\tau_2)r_1\right] c_1 .$$

Yes; $\frac{c_2}{c_1}$ depends upon taxes ($\tau_2$).

3. Ricardian Equivalence (or neutrality) means (see slides for details)....

4. No, it doesn't hold in this economy because

$J_2$ affects the optimal levels of consumption and savings of the representative household:

$c_1^*, c_2^*, a_1^*$.