The Inflation Game
Notes for E327 Fall 2001

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Abstract

A game theoretic model of the inflation-unemployment trade-off is analyzed. The game's players are the government and the market (private agents). The game's Nash equilibrium point determines the economy's level of expected inflation and the unemployment rate. It is shown that the expected rate of inflation is the actual rate chosen by the government in equilibrium and the level of unemployment equals the so-called *natural rate of unemployment*.

1. Introduction

The Phillips curve is an empirical relationship between the rate of inflation and the rate of unemployment. Since the 1960's economists have been tempted to interpret it as offering policy makers a way to lower unemployment by inflating the economy. Many theories in macroeconomics have been devised to support or criticize the idea that the U.S. economy's Phillips curve could and should be exploited to drive unemployment down at the expense of increased inflation.

Recent macroeconomic theories based on rational expectations models posit the private sector reacts to the target inflation rates chosen by the government and those private agents might alter their behavior in light of their expectations about government policy. Thus, the government must take private sector behavior into account when it formulates inflation policy. The rational expectations revolution took this one step further — it is assumed that private agents correctly anticipate the government's actions in equilibrium. The government knows this and takes it into account when deciding its policy. Game theory offers the conceptual framework for modelling these interactions between the government and the private market.

The purpose of these notes is to lay out a very simple model of the game played between private agents and the government when a Phillips curve relationship exists between inflation and unemployment. The game's Nash equilibrium outcome gives the economy's inflation and unemployment rates. The model is designed to illustrate how the Nash equilibrium concept might be used in a macroeconomic setting.

The model assumes that the private agents and the government interact in one period only. This is one, of many, unrealistic asssumptions that are imposed to make the presentation as simple as possible. The interested reader may consult the bibliography or any standard macroeconomic textbook for further motivation and detailed interpretations.

The model was developed by Kydland and Prescott [1] and refined by Stokey [3], [4]. My exposition follow's Sargent's [2] treatment.

2. The Inflation Game

There is a single period of time. There are two types of players. The government, which acts as a single entity, and private agents. I assume there are an infinite number of private agents (specifically, private agents can be indexed by the interval $[0, 1]$). The latter condition simply means that each private agent is small in relation to the total population and a single agent's actions, by themselves, have no consequences for aggregate behavior. Only the actions undertaken by **all** private market participants influence the game's outcome. I further assume that all private agents are identical — this implies that their actions are always identical and we can speak of a *representative private agent* because the actions taken by any single person are the same as those taken by all other private individuals.

The game's *timeline* is:

1. Private agents form expectations about the government's inflation policy.

2. The Government sets its actual inflation target.

3. The level of unemployment is determined by the Phillips curve (see below).

4. Payoffs for private agents and the government are calculated.

5. The game is solved by finding its unique Nash equilibrium.

It turns out that the model is very sensitive to the question of whether or not the government can commit to an inflation policy. The game considered here assumes the government cannot credibly commit to an inflation policy. This is modeled by letting the government move after private agents form their expectations.[1]

3. Players and Strategies

The modeling problem is to fully specify the players' strategies, payoffs, and the game's outcome(s).

3.1. The Government's Strategies

The government will be choosing a policy which determines the rate of inflation, denoted by y. We can simplify the model by assuming the inflation rate is the government's policy instrument even though the policy is usually carried out by employing a variety of instruments such as open market operations, setting reserve requirements, and fixing the Fed's discount rate. I assume that the government must select an inflation rate between -100 percent and 100 percent. That is, $y \in Y \equiv [-1, 1]$ defines the government's *feasible policy choices*. Notice that the inflation rate is measured as a percentage per year.

3.2. Private Agents Problems and Their Reaction Functions

Each private agent forecasts the inflation rate. This forecast is denoted by ξ. We can think of there being a distribution of forecasts across the continuum of private agents and compute the *public's average forecast*, which is denoted by x.

Private agents care about how good they forecast the inflation rate. Their payoffs depend only on the deviations of the actual inflation rate chosen by the

[1] In the case where the government moves first, one can find the so-called Ramsey equilibrium in which the government can commit to an inflation policy in advance of the private sector forming its inflation expectations. In doing so, the government operates under the assumption that it knows the ways in which private agents will react to its target inflation rate and takes this reaction into account in selecting its policy. See [2] for details. I am only concerned with the Nash problem in these notes.

government from their forecasted value. They also care about the overall inflation rate and prefer a lower inflation rate to a higher one. For simplicity, I assume that the magnitude of the deviations in their forecasts and the deviations of the actual inflation rate from a noninflationary state are all that matters to the private agents. Thus, I assume that their utility functions or payoff functions have the quadratic form:

$$u(\xi, x, y) = -\frac{1}{2}\left[(y-\xi)^2 + y^2\right]. \tag{1}$$

A private agent solves the problem

$$\max_{\xi \in Y} -\frac{1}{2}\left[(y-\xi)^2 + y^2\right] \text{ given } y.$$

Let $\xi(y)$ denote the solution to this maximization problem.

Remark 1. $\xi(y) = y$.

It is clear that the maximum value for a given y is larger the closer is ξ to y. Hence, private agents choose to set their forecasts to equal the given government inflation rate. That is, they correctly forecast the government's policy. Since **every** private agent forecasts the same inflation rate, it follows that $y = x$. The average forecast or public's forecast is the same as the government's expected choice, y.

We can summarize the private agents problems by saying that their optimization exercises produce their *best responses* or *reaction functions*, $\xi(y)$. For each possible forecast of the inflation rate, a private individual figures it is best (in terms of its reward function) to choose its forecast equal to the expected inflation rate, y.

In the following I take advantage of the assumption that all private agents have identical preferences and therefore share the same objectives. Thus, knowing the choice of an arbitrarily selected individual is the same as knowing the average choice since all agents choose identical strategies. Thus, it is enough to focus on the behavior of a *representative* private agent, one whose reaction function can be identified with the market's average response to its expectations of the government's policy decision. Hence, I will speak of the reaction function $y = x$ as the *market* or *public's reaction function* or its *best response function*. If the market expects the government will choose the inflation rate y, then those same private agents will make choices consistent with those expectations. That is, they will choose their optimal forecasts so that $y = x$.

I plot the private agent's reaction function $y = x$ in the (x, y)–plane in Figure 1. Evidently, its graph is the 45°-line.

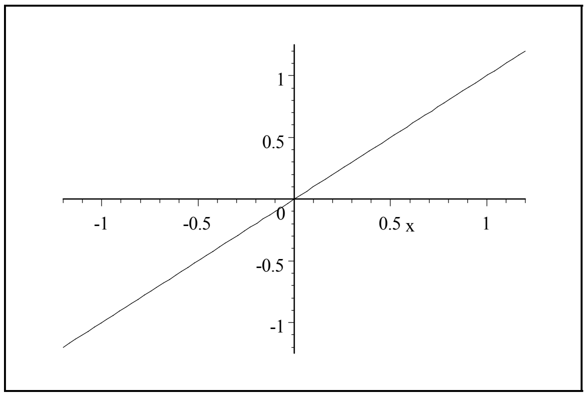

Figure 1. The private agent's reaction function is $y = x$. This states that their expected rate of inflation equals the rate of inflation they expect the government to choose when it moves.

3.3. The Government's Reaction Function

The government is assumed to care about the deviations of employment from a world without unemployment and deviations of the inflation rate from a noninflationary situation. For simplicity, this can be expressed as the quadratic objective:

$$G(U, y) = -\frac{1}{2}\left[U^2 + y^2\right], \tag{2}$$

where U is the unemployment rate.[2]

The inflation and unemployment rates are linked by a *Phillips curve*:

$$U = U^* - \theta(y - x), \tag{3}$$

where $\theta > 0$ is a parameter whose units are in workers per unit of inflation. The number U^* is known as the *natural rate of unemployment*.[3] The idea is

[2] This objective, like a private agent's objective, is overly simplistic. I defend these functional choices on the grounds that these quadratic forms allow an especially easy calculation of the various reaction functions or best response functions. The reaction functions turn out to be linear in the opponent's strategy.

[3] It is unreasonable to expect the rate of unemployment to be zero. Individuals change jobs. They are sometimes out of work for a period of time while looking for new jobs, and so on. The

that unemployment differs from the natural rate only when **surprise** inflation or deflation occurs.

Substitute (3) into (2) to obtain the government's reformulated objective function:

$$r(x,y) = -\frac{1}{2}\left[(U^* - \theta(y-x))^2 + y^2\right]. \qquad (4)$$

Now let $y = B(x)$ be the solution to $\max_y r(x,y)$, where the government takes the public's expectation, x, as given. The function $y = B(x)$ is the government's *best response* or *reaction function*. It can be shown that[4]

$$y = B(x) = \frac{\theta}{\theta^2+1}U^* + \frac{\theta^2}{\theta^2+1}x.$$

Notice that the government's reaction function is **linear** in the public's expectation of the inflation rate, x. This reaction function's slope is positive and smaller than one. Since there is a positive natural rate of unemployment, this reaction function's graph must cross the 45°-line in the (x,y)-plane. In fact, the reaction function cuts the 45°-line in one place and this occurs in the first quadrant — the solution is positive in each variable.

I plot this reaction function in Figure 2 for the case when $\theta = 1$ and $U^* = .04$. That is, the natural rate of unemployment is assumed to be 4 percent per year. Thus, Figure 2 shows $y = .04 + (1/2)x$.

$y = .04 + (1/2)x$

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natural rate of unemployment is supposed to be a measure of the minimal rate of uemployment that the economy supports even during good times as individuals must search for new jobs. The labor market is not entirely frictionless. It also measures the long-term structurally unemployed workers who are chronically out-of-work.

[4]Use calculus to compute the first-order condition $\partial r/\partial y = 0$ and solve it for y in terms of x.

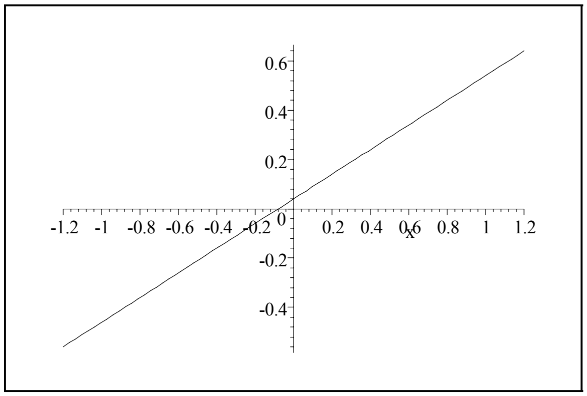

Figure 2. The government's reaction function for the case of a 4 percent natural rate of unemployment and $\theta = 1$.

4. The Nash Equilibrium Outcome

The Nash equilibrium for this game can be found at the point of intersection between the representative private agent's reaction and the government's reaction function. Let (x^N, y^N) denote a Nash equilibrium pair of strategies. It must satisfy the condition

$$y^N = x^N = B(x^N).$$

Notice that the Phillips curve (3) implies that $U^N = U^*$ in any Nash equilibrium That is, the Nash equilibrium level of unemployment always equals the natural rate. For any Nash equilibrium the private agent's assumptions about the government's inflationary target are correct. The government cannot fool or trick the private agents into believing the inflation rate will differ from its expected rate. Thus, an unexpected inflation or deflation is impossible in a Nash equilibrium.

It is clear that there can be only one Nash equilibrium strategy for the government and representative agent (see Figure 3 below) and it must be given by

$$y^N = \theta U^* = x^N.$$

I plot the equilibrium configuration by superimposing the reaction functions in Figures 1 and 2. If $\theta = 1$, then $y^N = U^*$ and this implies that $y^N = .04 = U^*$ in Figure 3. The figure has been drawn to "zoom" in on the model's equilibrium solution.

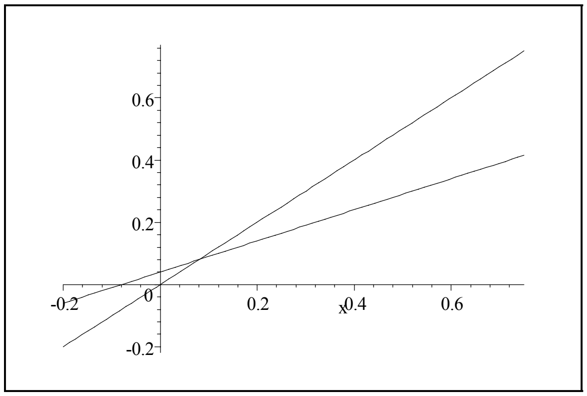

Figure 3. The intersection of the reaction functions derived from Figures 1 and 2 is the game's Nash equilibrium. This implies that $y^N = x^N = B(x^N)$ is a fixed point of the goverenment's reaction function.

5. Conclusion

The Nash equilibrium solution predicts a positive inflation rate and unemployment equals its natural rate. The government cannot induce a lower unemployment rate without creating a surprise inflation. The Phillips curve cannot be exploited for public policy goals in the economy's Nash equilibrium as the public's inflationary expectations are necessarily fulfilled.

References

[1] Finn Kydland and Edward C. Prescott, "Rules Rather than Discretion: the Inconsistency of Optimal Plans," *Journal of Political Economy*, June, Vol. 85 (3), 1977, pp. 473-491.

[2] Thomas J. Sargent, *The Conquest of American Inflation*, Princeton University Press, 1999.

[3] Nancy L. Stokey, "Reputation and Time Consistency," *American Economic Review Papers and Proceedings*, May, Vol. 79 (2), 1989, pp. 134-139.

[4] Nancy L. Stokey, "Credible Public Policy," *Journal of Economic Dynamics and Control*, October, Vol. 15 (4), 1991, pp. 627-656.