**ISCTE — INSTITUTO UNIVERSITÁRIO DE LISBOA**

**BA in Economics**

**Modern Macroeconomics**

**Sample Midterm test — SOLUTIONS**

**20 March– 2012** **Duration: 1.30 h**

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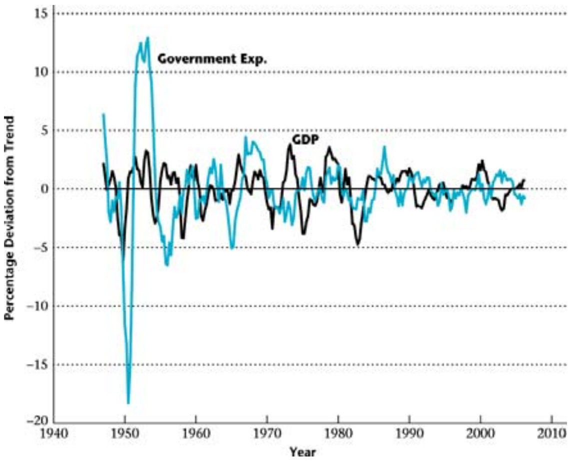
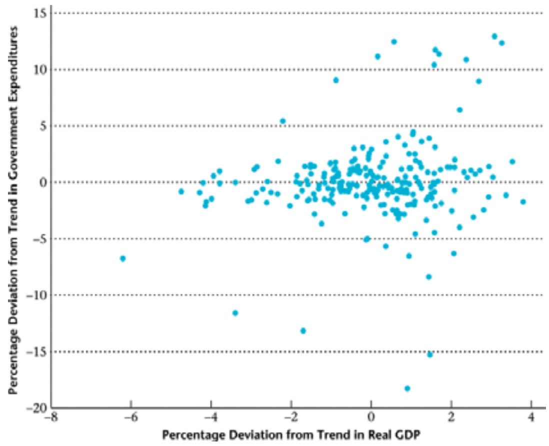
**A.** Considering the main analytical tools normally used to analyze the main characteristics of business cycles, answer the following questions:

1. Describe the dynamic response in a stochastic process that is usually referred to by "impulse response functions". Why are they relevant for modern macroeconomics? **(25 points)**

2. Consider the next two figures, which show the relationship between the deviations from the long term trend of GDP ($Y$) and government expenditures ($G$), for the US economy in post World War II period.




As far as the main stylized facts of the business cycles are concerned, what do you conclude about the behavior of these two macro variables? **(25 points)**

**Answers.**

The answer to each one of these two questions should be based upon the material that was covered in classes. In some cases, like these two questions, the answer does not depend on any kind of interpretation of the problem, or on any derivation of results. It is just a mere direct reproduction of what was explained in classes. There may be some cases where you are asked to explain some phenomenon by applying the theory (or the techniques) that you were taught. But this is not what we have in the two questions here.

**Question A1**. It's about impulse response functions (IRF). The answer should include:

1. A definition (in your own words) of what an IRF is: see slides
2. How can such dynamics be produced? See slides (concentrate on the AR(1) case, because it's simpler)
3. Why is an IRF useful for modern macroeconomics: we can see how the introduction of shocks to a particular economic or financial process affects its dynamics, and afterwords we can compare the output of this process with data from reality.

**Question A2**. This question requires knowledge about the main concepts of short term business cycles: (i) countercyclical variables, pro-cyclical, and acyclical; (ii) leading, lagging and coincident variables, and (iii) volatility.

1. Is $G$ procyclical, or ...? By just looking at the left panel it is difficult to answer about that. The right panel (a cross plot, not a time series) gives us the answer: $G$ is acyclical, because correlation is neither positive nor negative.
2. Given the two panels above, it seems difficult to tell whether $G$ is leading, lagging or a coincident variable. We need some extra information to provide such an answer with some confidence in our arguments.
3. As far as volatility is concerned, it seems clear that $G$ was more volatile than $Y$ up to the mid 1970s. But the evidence is not as clear cut as before, for the period post mid 1970s. For this period we need more information. We need to know what is the variance or the standard deviation about each series for this latter period in order to answer the question properly.

**B.** Consider an economic process that can be described by the following system

$$\begin{aligned} x_{t+1} &= 2 + 0.8x_t + 1.4y_t \\ y_{t+1} &= 10 + 0.5y_t \end{aligned}$$

1. Does this system has a steady state (or a fixed point)? Explain. **(10 points)**
2. If there is a fixed point, is this unique or are there multiple equilibria? **(10 points)**

3. What is the type of stability in this process? Justify your answer. **(15 points)**
4. Assume now that the second equation is written as

$$y_{t+1} = 10 + 0.5y_t + c_t$$

where $c_t$ is an external shock normally described as "white noise": $c_t\tilde{\ }N(0,1)$. Does this shock produce any relevant changes to the previous results. Explain. **(15 points)**

**Solution.**

$$\begin{aligned} \bar{x} &= 2 + 0.8\bar{x} + 1.4\bar{y} \\ \bar{y} &= 10 + 0.5\bar{y} \end{aligned}$$

1. Solution is: $\{[\bar{x} = 150.0, \bar{y} = 20.0]\}$

2. Unique solution, therefore the fixed point is unique

3. $A = \begin{vmatrix} 0.8 & 1.4 \\ 0 & 0.5 \end{vmatrix}$, eigenvalues of $A$: $0.8, 0.5$;the fixed point is stable, both eigenvalues are lower than 1 in absolute value.

4. Because $c_t$ is white noise, the long term equilibrium value of $y$ and, consequently, that of $x$ are not altered. The only difference now is that both $\bar{x}$ and $\bar{y}$ over time revolve around the long term equilibrium values of $\{[\bar{x} = 150.0, \bar{y} = 20.0]\}$ in a usual random manner.

**C.** Assume that a certain economic process can be explained by the following nonlinear difference equation:

$$x_{t+1} = f(x_t) = 4 + 5x_t - x_t^2$$

1. How many fixed points do we have in such a process? Explain. **(10 points)**
2. Represent the expression that provides the linear approximation in the neighborhood of each fixed point. **(10 points)**
3. Which of those fixed points are stable and unstable? **(10 points)**
4. Imagine now that the process starts at the initial value: $x_0 = 2$. What happens over time? **(10 points)**
5. And what would happen if the initial value were $x_0 = -2$. What happens over time? **(10 points)**

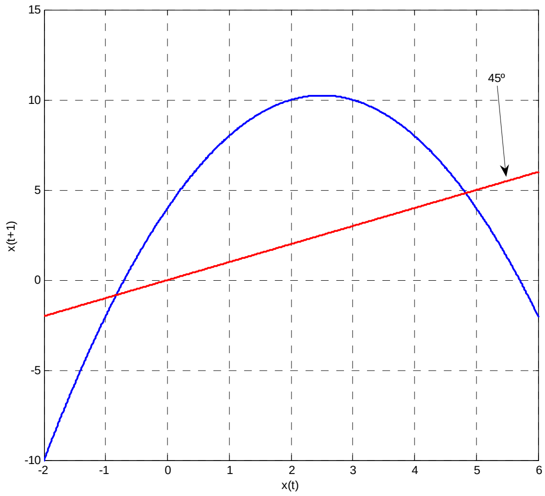



Figura 1: Figure not expected to be included in your answer.

**Solution.**
1. Solving

$$\begin{aligned} x_{t+1} &= x_t = \bar{x} \\ \bar{x} &= 4 + 5\bar{x} - \bar{x}^2 \\ 4 + 4\bar{x} - \bar{x}^2 &= 0 \end{aligned}$$

Solution is: $\{[\bar{x} = 4.8284], [\bar{x} = -0.82843]\}$.(See Figure 1 above. Notice that the answer is not expected to include such a figure). So we have two fixed points.

2. The linear approximation is given by

$$f(x_t) \approx f'(\bar{x})(x_t - \bar{x}) + f(\bar{x})$$

Let's first calculate the first derivative $f'(x_t)$ Therefore, as

$$f'(x_t) = 5 - 2x_t$$

For $\bar{x} = 4.8284$,we get

$$f'(\bar{x} = 4.8284) = 5 - 2 \times 4.8284 = -4.6568$$

For $\bar{x} = -0.82843$,we get

$$f'(\bar{x} = -0.82843) = 5 - 2 \times (-0.82843) = 6.6569$$

Then the linear approximation at each fixed point is for $\bar{x} = 4.8284$ given by,

$$\begin{aligned} f(x_t) &\approx -4.65(x_t - 4.828) + 4.828 \\ &\approx 27.278 - 4.65x_t \end{aligned}$$

and for $\bar{x} = -0.828\,43$

$$\begin{aligned} f(x_t) &\approx 6.\,656\,9(x_t - (-0.828\,43)) - 0.828\,43 \\ &\approx 4.686 - 6.\,656\,9x_t \end{aligned}$$

3.The two fixed points $\{[\bar{x} = 4.\,828\,4], [\bar{x} = -0.828\,43]\}$, both are unstable given that $f'(\bar{x})$ is larger than 1 in absolute value in each case.

4. The dynamics will diverge from any one of the two fixed points.

5. Identical answer to the previous one.

**D.** Assume that the utility of a representative consumer is a function of the level of consumption $(c_t)$

$$u(c_t) = \ln c_t$$

Her/his objective is to maximize intertemporal utility discounted by a factor $\beta$

$$\max u(c_t) + \beta \cdot u(c_{t+1})$$

subject to the two usual constraints:

$$\begin{gathered} c_t + a_{t+1} \leq w_t \\ c_{t+1} \leq (1 + r_{t+1})a_{t+1} + w_{t+1} \end{gathered}$$

1. Show that this utility function satisfies the basic hypotheses of the theory of a rational consumer. **(5 points)**
2. Derive the Euler equation associated with this utility function. **(10 points)**
3. Determine the optimal consumption levels for each period, as well as the optimal savings level, by considering the following parameter values: $w_t = 100, w_{t+1} = 130, r_{t+1} = 4\%$, and $\beta = 0.9$. **(15 points)**
4. Now take into account that this consumer is not allowed to borrow more that 15% of his/her wages at $t$. Do you consider this consumer to be financially constrained? Explain graphically the new equilibrium. **(10 points)**
5. In terms of social welfare, which situation is better: the initial situation or the new one with the financial constraint? Explain. **(10 points)**

**Solution.**
1. $u'(c_t) = 1/c_t > 0, \;\; u''(c_t) = -1/c_t^2 < 0$

2. This is a totally standard exercise. So no relevant explanation is required here (see slides). Write the Lagrangian, take the first order conditions and you will get at the Euler equation

$$c_{t+1} = \beta(1 + r_{t+1})c_t$$

3. The intertemporal consolidated budget constraint is given by

$$\begin{aligned} c_t + \frac{c_{t+1}}{1 + r_{t+1}} &= w_t + \frac{w_{t+1}}{1 + r_{t+1}} \\ c_t + \frac{c_{t+1}}{1.04} &= 100 + \frac{130}{1.04} \end{aligned}$$

Using the Euler equation and the consolidated constraint we will get

$$\begin{aligned} c_{t+1} &= 0.9(1.04)c_t \\ c_t + \frac{c_{t+1}}{1.04} &= 100 + \frac{130}{1.04} \end{aligned}$$

Solution is: $\{[c_t = 118.42, c_{t+1} = 110.84]\}$ .Now the optimal level of savings can be easily obtained

$$\begin{aligned} a_{t+1} &= w_t - c_t \\ &= 100 - 118.42 = -18.42 \end{aligned}$$

4. Yes, he/she is financially constrained. His wage at $t$ is 100, and he likes to consume at $t$ 118.42. But if he is allowed to borrow no more than 15% of $w_t$, then the maximum amount of $c_t$ will be 115, not 118.4.

5. Talking about the social welfare of our consumer, yes, the initial situation $(c_t = 118.42)$ is better in terms of her social welfare than the new one ($c_t = 115$). This occurs because the MRS is higher than the relative price of intertemporal consumption. The case is that our consumer favours more current consumption than future consumption, and the financial constraint interferes with this individual taste, forcing our consumer to change her optimal intertemporal consumption basket. (Reproduce the standard figure of a financially constrained agent).